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Physics - From Stargazers to Starships

Stern

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Chapter 1

Stargazers and Sunwatchers

1.1 Watchers of the Heavens

Early stargazers — especially the priests of Egypt and Babylon, semi-desert countries where skies are rarely clouded — were fascinated by the star-studded canopy which seemed to arch overhead, and by the daily cycle of the Sun, which seemed supernatural, beyond understanding. The ancient author of Psalm 19 wrote:

> The heavens declare the glory of God,
> And the firmament showeth His handiwork;
> Day unto day uttereth speech,
> And night unto night revealeth knowledge;
> There is no speech, there are no words,
> Neither is their voice heard.
> Their line is gone out through all the earth,
> And their words to the end of the world.
> In them He has set a tent for the Sun,
> Which is like a bridegroom coming out of his chamber;
> And rejoiceth as a strong man to run his course.
> His going forth is from the end of the heaven,
> And his circuit unto the ends of it;
> And there is nothing hid from the heat thereof.

East, West, South, and North

Imagine you were one of the early Babylonian skywatchers! You live on a plain, and as far as you can see, the world around you is absolutely **flat** (only careful observations of the surface of the ocean suggest anything different — See the chapter "The Round Earth and Columbus" in the *From Stargazers to Starships FlexBook* on www.ck12.org). Your view is limited by the **horizon**, an imaginary line all around you at a distance of a few miles, or whatever units Babylonians used.

Observing day after day, you note that the Sun always rises from roughly the same direction, which you name **east**. It sets in the opposite direction, and that will be **west**. In between the Sun rises in a long arc, and is furthest from the horizon halfway between its rising and setting, in a direction you call **south**. Finally, the direction opposite south will be **north**.

When the Sun is near the horizon, shortly after sunrise or before sunset, a vertical pole or post casts a **long** shadow. At the highest point in the Sun's motion, when it is in the south, the shadow is at its shortest. The time when this happens is halfway between sunrise and sunset, and we call it **noon** or maybe "noon by the Sun," because "noon by the clock" may differ. After noon shadows again grow longer, as the Sun descends towards the horizon.

Because the shadow always points **away** from the Sun:

- At sunrise, with the Sun in the east, it points to the west.
- At noon, with the Sun in the south, it points north.
- At sunset, with the Sun in the west, it points to the east.

That is the principle of the **sundial**, discussed in the chapter "Making a Sundial" in the *From Stargazers to Starships FlexBook* on www.ck12.org.

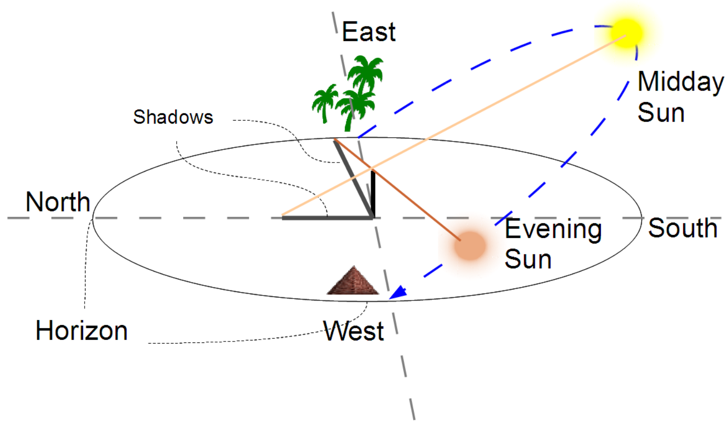

Figure 1.1: The Sun shortest shadow is due north at noon, while the evening sun creates a longer shadow due east.

Suppose you watch the Sun rise and set day after day. Using as markers features on the horizon — trees, houses, etc. — you soon realize that the points where Sun rises and sets are not always the same, but shift week after week. On the other hand, the direction of **south**, where the Sun is **highest** above the horizon **does not change**, and neither does that of **north**, of the shortest shadow of the day. Because those directions are fixed, it is best to choose as the 'true' east and west those directions which are perpendicular to north-south. Only **twice** each year are sunrise and sunset exactly in those directions, but they help measure and understand what happens in the rest of the year.

Seasons of the Year

Even in Babylon the year has seasons–winters are cool, summers dry and very hot. As already noted, twice a year, halfway between summer and winter, the Sun rises **exactly** in the east (as defined above), and sets **exactly** in the west (well, nearly exactly, in both cases). We now know that on the days when this happen, day and night are very nearly equal in length, and that time of year is therefore called **equinox.**

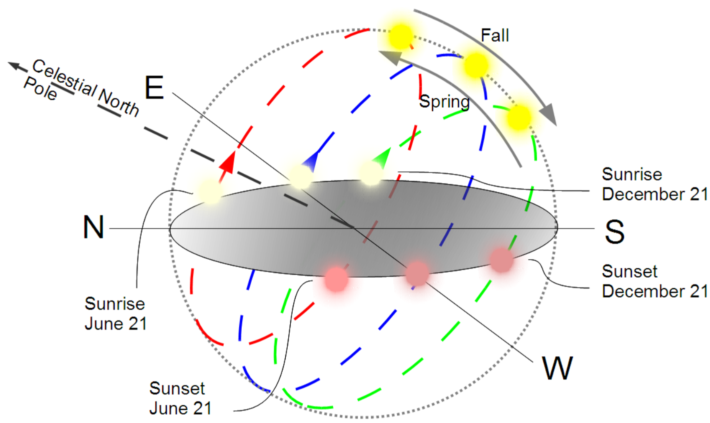

Figure 1.2: In summer, the Sun's path is longest, and so are the days. In winter, the Sun's path is shortest, and so are the days.

One equinox happens in the fall ("autumnal equinox") and one in the spring ("vernal equinox," "ver" is Latin for spring).

As fall advances towards winter, the location of sunrise moves south, as does the location of sunset. The steepness of the curve traced by the Sun does not change, nor does the rate ("speed") with which the Sun appears to move along it, but the **length** of the curve changes, it becomes shorter. Around December 21 –the "**winter solstice**" halfway between the equinox dates (typically, September 23 and March 21) sunrise and sunset are as far south as they can go (at any one location). As a result, the Sun has its shortest path for the year, the day is at its shortest and night is at its longest. Other days of that season are short, too, which is one reason for the colder weather in winter.

After that the points of sunrise and sunset migrate northward again, and days get longer. This migration continues past equinox (when it is at its fastest), and the Sun crosses the horizon furthest northwards around June 21, the "**summer solstice**" (celebrated in some cultures as "midsummer day"), longest day of the year with the shortest night. After that days get shorter again as sunset and sunrise migrate south again. The long days of summer, of course, match the warmer summer weather. The reason for this behavior will be described in the chapters "The Ecliptic" and "Seasons of the Year" in the *From Stargazers to Starships FlexBook* on www.ck12.org.

1.2 Elevation of the Sun

The length of the day is not the only reason summers are hot and winters cold. Another is the **elevation of the Sun above the horizon.** When the Sun is near the horizon, not only are the shadows which it casts stretched to greater length, so is its illumination. Any beam of sunlight then spreads out along a greater distance on the ground, diluting the heat given to any area. The noontime Sun in winter is low in the sky, and its heating is less pronounced, while the summer Sun can be almost overhead, heating the ground much more effectively. This is further discussed in the chapter "The Angle of the Sun's Rays" in the *From Stargazers to Starships FlexBook* on www.ck12.org.

Babylonian priests, who tracked these regular changes of sunrise and sunset, soon realized that they provided an accurate way of measuring the passage of the seasons. They counted the days between solstices and equinoxes, and from this the first **calendar** was born. That was a great help to farmers, telling them when to prepare for sowing, when to expect seasonal rain, and in Egypt, when to expect the annual flood of the river Nile, which replenished the land. As will be described in the chapter "The

Calendar" in the *From Stargazers to Starships FlexBook* on www.ck12.org, other cultures also had their stargazers and developed calendars of their own, probably in much the same way.

Image Sources

(1) Alex Zaliznyak, David Stern. *Shadow Diagram*. CC-BY-SA 3.0.

(2) Alex Zaliznayk, David Stern. *Path of the Sun*. CC-BY-SA 3.0.

Chapter 2

The Celestial Sphere

The Sun rules by daytime sky, but at night, especially if the Moon does not shine, the show belongs to the stars. Bright and dim, randomly distributed across the sky, with odd formations that catch the eye, their number seems huge. To ancient observers it seemed as if Earth was at the center of a giant star-studded "**celestial sphere**," which reinforced the belief, held for thousands of years, that **we** are at the center of the universe.

If you watch stars throughout the night, you will see that most of them also rise to the east of you and set west of you, like the Sun and Moon. Indeed, the entire celestial sphere seems to rotate slowly — one turn in 24 hours — and since half of it is always hidden below the horizon, this rotation constantly brings out new stars on the eastern horizon, while others to disappear beneath the western one.

We of course know that it is **not** the universe that rotates around us from east to west, but **our Earth** is the one rotating, (from west to east–see note at end). But it is still convenient to talk about "the rotation of the celestial sphere." That could also make the sky rotate the way it is observed to do.

Note: The text above — and sections that follow — gives the period of rotation of the Earth as 24 hours. **That is not exactly true**: 24 hours is the mean length of a **solar day**, the average time that passes from noon to the next noon. Noon is always defined by the position of the Sun — when it passes exactly to the south (to viewers in Europe and the US, at least), and is at its greatest distance from the equator.

Using the Sun for reference, however, gives a **shifting** reference point in the sky. Between one noon and the next, the Sun too **moves** slightly in the sky, as part of its annual circuit around the celestial sphere, discussed in the next section, on the ecliptic (See the chapter "The Ecliptic" in the *From Stargazers to Starships FlexBook* on www.ck12.org). We could instead use **some star** as reference point, since stars keep fixed positions on the celestial sphere (see further below): for instance, define as "**sidereal day**" (sidereal — related to stars) the time between one passage of Sirius (the brightest star) to the south, and the next passage. **That** would be the **true rotation period** of the Earth, shorter than 24 hour by nearly 4 minutes — more accurately, 235.9 seconds.

> (**If you wish to calculate the difference**: 24 hours are equal to 86400 seconds, and the average year contains 365.2422 solar days (See the chapter "The Calendar" in the *From Stargazers to Starships FlexBook* on www.ck12.org, where this point is also discussed). Actually, however, the Earth completes 366.2422 rotations in that time, so the real rotation period is just (365.2422/366.2422) of 86400 seconds. You should be able to figure out the rest.)

Most stars keep fixed positions relative to each other, night after night. The eye naturally groups them into patterns or **constellations** ("stella" is Latin for star), to which each culture has given its own names.

The names we use come from the ancient Greeks and the Romans, e.g. **Orion** the hunter, accompanied by his two faithful dogs nearby. Other names evoke animals, whose Latin names are used — **Scorpio** the scorpion, **Leo** the lion, **Cygnus** the swan, **Ursa Major** the Big Bear (better known as the "big dipper") and so forth.

The Sun slowly moves through this pattern, circling around it once a year, always along the same path among the stars ("the ecliptic"). The ancients distinguished 12 constellations along this path, and since most are named for animals, they are known as the **zodiac**, the "circle of animals." The Sun spends about one month inside each "sign of the zodiac." The Moon moves close to the Sun's path, but only takes about a month, and a few conspicuous stars also move near it, the planets. We will come back later to all these: all other celestial objects are firmly placed and do not move, forming the "firmament."

Like the globe in the drawing, the sphere of the sky has two points around which it turns, points that mark its axis —the **celestial poles**. Stars near those poles march in daily circles around them, and the closer they are, the smaller the circles (they **do not** rise and set). At any time, only **half** the sphere is visible: it is as if the flat ground on which we stand sliced the celestial sphere in half — the upper half is seen, the lower half is not. Because of that, only one pole is seen at any time, and for most of us, living north of the equator, that is the north pole.

> (If you mount a camera on a dark night in a way that the pole is in the middle of its field of view, open the shutter and take a time exposure, the image of each star will be smeared into part of a circle, and all the circles will be centered on the pole. Go to `http://antwrp.gsfc.nasa.gov/apod/ap980912.html` to see such a picture.)

Just as the globe of the Earth has an equator around its middle, halfway between the poles, so the sphere of the sky is circled by the **celestial equator**, halfway between the celestial poles. As the sky rotates, stars on the equator trace a longer circle than any others.

Of course, we know well (as the priests in Babylon didn't) that the stars are not attached inside a huge hollow sphere. Rather, it is **the Earth** which rotates around its axis, while the stars are so distant that they seem to stand still. The final effect, however, is the same in both cases. Therefore, whenever that is convenient, we can still use the **celestial sphere** to mark the positions of stars in the sky.

2.1 Polaris, the Pole Star

By pure chance, a moderately bright star is seen near the northern celestial pole–Polaris, the pole star (or north star). Polaris is not exactly at the pole, but its daily circle is very small and for many purposes one can assume it is at the pole, a pivot around which the entire sky rotates.

All this looks much clearer if one remembers that it is the **Earth** that rotates, not the sky. The axis around which the Earth spins points in a certain direction in the sky, and that is also the direction of the pole star (or more accurately, the northern celestial pole). As the Earth turns, even though the observer moves with it (for instance, from point B in the drawing to point A), that direction always makes the same angle with the horizon and is always to the north. Hence the pole star is always in the same spot — north of the observer, and the same height above the horizon.

If on a clear night you find yourself **lost** in the wilderness or at sea, the pole star can tell you where north is, and from that you easily deduce east, west and south. Any other star is unreliable for determining direction — it will move across the sky, and may even set — but not this one. For instructions on finding the pole star at night, go to the chapter "Finding the Pole Star" in the *From Stargazers to Starships FlexBook* on www.ck12.org.

The closer you are to the equator, the closer is the pole star to the horizon, and **at** the equator (point C) it

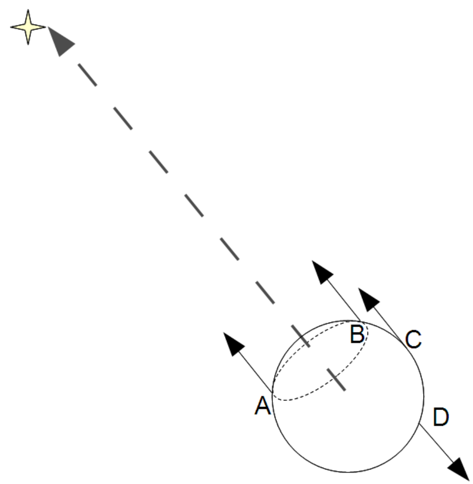

Figure 2.1: (Not to scale) Polaris lies near the Earth's rotation axis.

is **on** the horizon, and probably not easy to see. Further south, at points such as D, it is no longer visible, but now you can see the southern pole of the sky. Unfortunately, no bright star comparable to Polaris marks that position. The existence of a bright star near the north celestial pole is just a lucky accident, and as will be seen, it wasn't always so, and will not be a few thousand years from now.

2.2 The Mounting of a Telescope

As the drawing above makes clear, during the night we view the pole star from different positions (such as A and B). This however makes no noticeable difference in its place in the sky, because it is so distant from us. If the Earth rotated not around its axis but along a **parallel** line through A or B, the sky would not appear any different.

To the eye the rotation of the sky is very, very slow (it is most noticeable when the Sun or Moon are rising or setting). A telescope however greatly magnifies the rotation rate, and any star observed with it quickly drifts to the edge of the field of view and then disappear, unless the direction of the telescope is constantly adjusted. That is usually done automatically, by turning the telescope around an axis parallel to the Earth's rotation, for as explained above, a parallel shift does not change the apparent rotation of the stars.

To make such an adjustment easy, an **astronomical telescope** (pictured above) is mounted very differently from a **surveyor's telescope** (a "theodolite," pictured below). A theodolite usually has two axes–one allows it to scan all horizontal directions over 360 degrees, while the other adjusts its elevation and allows it to set its sights on reference marks higher than the viewer, such as mountaintops. On the other hand, a telescope for viewing stars (above) also has two perpendicular axes, but the main one (the "**equatorial axis**") is slanted to point at the pole star and is therefore parallel to the Earth's axis. As the celestial sphere rotates, a clockwork (or in cheap telescopes, the hand of the observer on a suitable knob) turns the telescope at a matching rate, keeping the same stars in the field of view.

2.3 Planets and the Zodiac

Not all stars keep fixed positions on the sphere of the heavens. Even early sky-watchers noted that a few moved about: the ancient Greeks called them "**planets**", which translates to wanderers. The names we

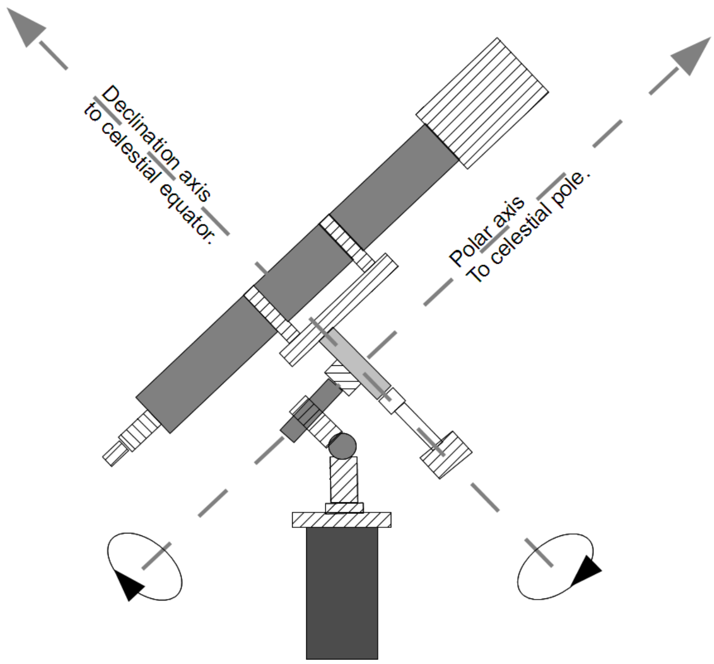

Figure 2.2: The equatorial mounting of a telescope. To track a star it is only necessary to rotate the telescope around its polar axis.

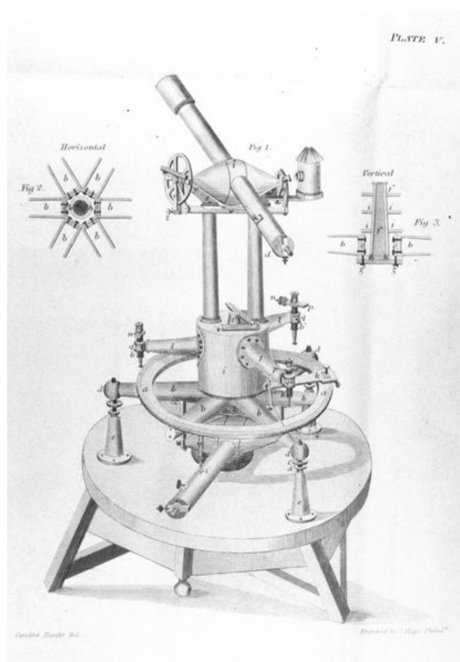

Figure 2.3: An old surveyor's telescope (theodolite).

use today came from the Romans, who named them after their chief gods — Mercury, Venus, Mars, Jupiter and Saturn. Mercury and Venus are always close to the Sun and can only be seen shortly after sunset or before sunrise: Mercury is so close that most of the year it cannot be seen at all, because the bright sky drowns out its light. Venus is brighter than any other star (with appropriate conditions and looking right at it, you can see it even in the daytime) and Jupiter takes second place.

2.4 Note on Earth's Rotation

You can demonstrate that the rotation is **west to east** by using an apple or some other fruit to represent the Earth. Hold it with the stem vertical–that would be the axis of the Earth, north pole on top–and mark two points in the northern half — **New York** and slightly **clockwise** from it, (= westward) **San Francisco**. You can use a lamp as your "Sun," or imagine having a lamp somewhere nearby.

When it is noon in New York, the Sun is almost overhead above "New York," but it is still only 9 in the morning in "San Francisco." Three hours later, the Earth has rotated and **now** it is noon at "San Francisco," with the Sun close to overhead. To get to this position, San Francisco must rotate to the position New York was in–from west to east.

Image Sources

(1) Caroline Hassler. *Theodolite*. Public Domain.

(2) David Stern, Alex Zaliznyak. *Polaris*. CC-BY-SA 3.0.

(3) David Stern, Alex Zaliznyak. *Mounting Telescope*. CC-BY-SA 3.0.

Chapter 3

Finding the Pole Star

Two bright constellations occupy opposite sides of the pole star — the **Big Dipper** and **Cassiopeia**. As the celestial sphere rotates (or appears to rotate), these constellations also march in circles around the pole . Depending on the hour of the night and the day of the year, one or the other may be low near the horizon where it is barely seen, or even hidden below the horizon. But when that happens the **other** constellation is sure to be high in the sky, where (weather permitting) it is easily seen.

3.1 The Big Dipper

The Big Dipper consists of 7 bright stars, forming a dipper, a small pot with a long handle. In England it is often called "the plough" (spelled "plow" in the US), and fugitive slaves before the Civil War knew it as "the drinking gourd," a signpost in the sky pointing the way north to safety, to Canada where slavery was outlawed. Astronomers name it "Ursa Major," Latin for "the big she-bear," and some other languages also refer to it as the Big Bear. In Greek, bear is "Arktos," and hence the far-north region where this constellation is usually overhead became known as "the Arctic."

When the territory of Alaska in 1926 decided to create a flag of its own, it asked citizens to submit proposed designs for the new flag. The winning design was that of Benny Benson, age 13, and is reproduced here (**more about him — see below**). It shows the 7 stars of the Big Dipper and Polaris, the north star. When Alaska became a state, this became the state flag.

The flag also shows how the north star can be found. Imagine a line connecting the two stars at the front of the "dipper," continue it on the side where the dipper is "open" to a distance 5 times that between the two stars (the flag shortens this a bit!), and you will arrive at (or very close to) the pole star. Because of their role in locating Polaris, these two stars are often called "the guides." And by the way — the last-but-one star in the handle of the "dipper," named Mizar by Arab astronomers, is a double star, whose components are readily separated by binoculars — or, some say, by very sharp eyes during good viewing conditions.

3.2 Cassiopeia

Cassiopeia was a queen in Greek mythology, and the constellation named for her is shaped like the letter W. Polaris is above the first "V" of this letter. If you draw a line dividing the angle of that "V" in half and continue along it, you will reach the vicinity of Polaris.

The name of Cassiopeia's husband, King Cepheus, goes with a nearby constellation, above the other "V" (the brighter one), but Cepheus is nowhere as striking as Cassiopeia. Her daughter Andromeda has another

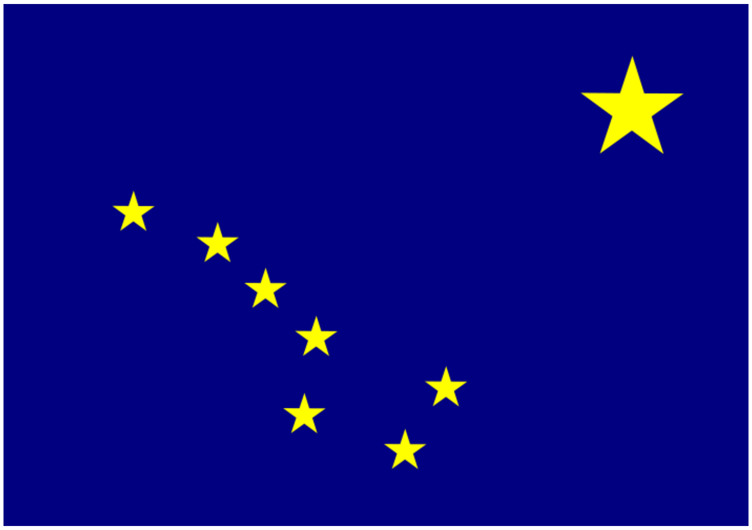

Figure 3.1: The flag of Alaska.

constellation, framed by a big undistinguished rectangle of four stars. An unremarkable constellation to the eye — but it contains a large galaxy, our nearest neighbor in space (not counting two dwarf galaxies in the southern sky), one which seems to resemble ours in size and shape.

Ursa Minor, the "Small Bear" or "Little Dipper&rdquo is a constellation somewhat resembling the Big Dipper, and Polaris is the last star in its tail. The "dipper" itself faces the tail of the Big Dipper, so that the two "tails" (or "handles") point in opposite directions. The two front stars of the "little dipper" (quite smaller and more square than the big one) are fairly bright, but other stars are rather dim and require good eyes and a dark sky.

3.3 Further Exploration

Benny Benson was of mixed Swedish-Alutiiq parentage and grew up in the Aleut islands. He was placed in the Jesse-Lee Memorial Home for Orphans in Unalaska, Aleut islands, and later moved to Seward, Alaska, where he was attending the 7th grade when he proposed his flag design. He is honored with a monument at the end of 3rd Ave. in Seward.

Later Benny became an airplane mechanic and lived on Kodiak. Throughout his life he made miniature Alaska flags and some are displayed in various public places. For more visit http://en.wikipedia.org/wiki/Benny_Benson.

Image Sources

(1) Chris Hilker, openclipart.org. *Alaska's Flag*. Image in Public Domain.

Chapter 4

The Ecliptic

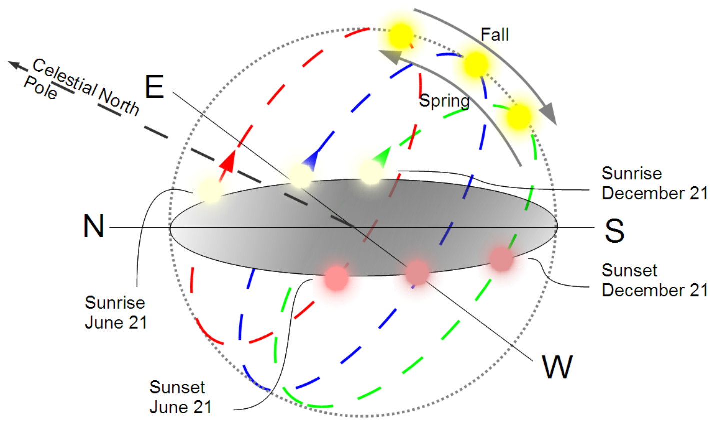

Figure 4.1: In summer, the Sun's path is longest, and so are the days. In winter, the Sun's path is shortest, and so are the days.

4.1 Signs of the Zodiac

Even though the planets move on the celestial sphere, they do not wander all over it but are confined to a narrow strip, dividing it in half. Stars along that strip are traditionally divided into the 12 constellations of the **zodiac**. The name, related to "zoo," comes because most of these constellations are named for animals - Leo the lion, Aries the ram, Scorpio the scorpion, Cancer the crab, Pisces the fish, Capricorn the goat and Taurus the bull.

At any time, the Sun is also **somewhere** on the celestial sphere, and as the Earth turns, it rises and sets the same way as stars do.

Like the planets, the Sun, too, moves around the zodiac, making one complete circuit each year. Every month it covers a different constellation of the zodiac, which is the real reason why those constellations are 12 in number. Of course, during that month, this particular constellation is not seen, because the sky near the Sun is too bright for its stars to be seen (except, very briefly, during a total eclipse of the Sun).

One can however figure out where the sun is on the zodiac (as ancient astronomers have done) by noting which is the **last** constellation of the zodiac **to rise** ahead of the Sun, or the **first to set** after it. Obviously, the Sun is somewhere in between. In this manner each month-long period of the year was given its "sign of the zodiac."

Astrologers, who believe that stars mysteriously direct our lives, claim it makes a great difference "under what sign" a person was born. Be aware, however, that the "sign" assigned to each month in horoscopes is not the constellation where the Sun is in that month, but where it would have been in ancient times. The difference is discussed in the section on the precession of the equinoxes (See the chapter "The Precession" in the *From Stargazers to Starships FlexBook* on www.ck12.org).

4.2 The Ecliptic

The path of the Sun across the celestial sphere is very close to that of the planets and the moon. After clocks became available, it was a relatively straightforward job for astronomers to relate the path of the Sun in the daytime to the one of stars at night, and to draw it on their star charts. Because of its relation to eclipses, that path is known as the **ecliptic**.

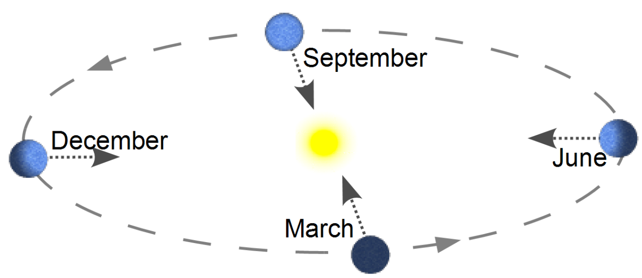

Figure 4.2: The orbit of the Earth around the Sun. This is a perspective view, the shape of the actual orbit is very close to a circle.

4.3 The Planets and the Moon

Planets seen in the sky are always near the ecliptic, which means that their orbits are never too far from the plane of the ecliptic. In other words, **the solar system is rather flat**, with all its major parts moving in nearly the same plane.

What about the connection between "ecliptic" and eclipses?

The moon's orbit cuts the ecliptic at a shallow angle, around 5 degrees, which means that on the celestial sphere the Moon, too, follows a path through the zodiac. Half the time the Moon is north of the ecliptic, half the time south of it. If the shadow of the moon hits the Earth, the Sun is eclipsed in the shadow area; if on the other hand the shadow of the Earth covers the moon, the moon goes dark and we have an eclipse of the moon.

Either of these can **only** happen when the Sun, Earth and Moon are on the same straight line. Since the Sun and Earth are in the plane of the ecliptic, the line is automatically in that plane too; if the moon is also on the same line, it must be in the plane of the ecliptic as well.

It takes close to a month for the Moon to go around the Earth ("month" comes from "Moon") and during that time its orbit crosses the ecliptic **twice**, as it goes from one side to the other. At the time of crossing, the Sun may be **anywhere** along the ecliptic; usually it is **not** on the Earth-Moon line, and therefore an eclipse usually does **not** take place. Occasionally, however, it is on that line or close to it. If it then happens to occupy exactly the **same** spot on the celestial sphere, we get an eclipse of the Sun, because the moon is then between us and the Sun. On the other hand, if it occupies the spot exactly **opposite** from that of the Moon, the Earth's shadow falls on the Moon and we have an eclipse of the Moon.

Exploring Further

http://antwrp.gsfc.nasa.gov/apod/ap960921.html is a NASA webpage showing three planets and the Sun lined up along the ecliptic.

Image Sources

(1) Alex Zaliznyak, David Stern. *Path of the Earth*. CC-BY-SA 3.0.

(2) David Stern, Alex Zaliznyak. *Path of the Sun*. CC-BY-SA 3.0.

Chapter 5

Making a Sundial

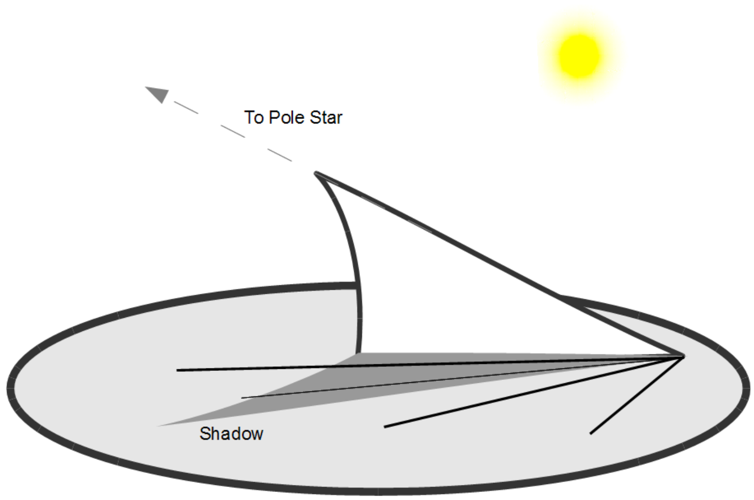

Figure 5.1: A Sundial

> "Some people can tell what time it is by looking at the sun. But I have never been able to make out the numbers."
>
> (Attributed to an essay by a student in elementary school.)

The simplest sundial is a **vertical stick** rising from a flat horizontal surface.

As the Sun rises, passes the highest point in its path (at noon and to the south, in the northern hemisphere) and sets, **the shadow rotates** around the stick in a clockwise direction, and its position can be used to mark time. Indeed, it has been claimed that the "**clockwise**" direction in which the hands on a clock rotate was chosen for this reason.

A sundial with a **vertical** pointer ("**gnomon**") will indicate noon correctly when its shadow points north or south. [North in northern middle latitudes, south in southern ones, while near the equator it can be either way, depending on season.] However, the direction of the shadow at some other time of the day may depend on the season–its value in summer, when the Sun's path is high, may differ from what it is in winter, with Sun low above the horizon.

Such a sundial will however work equally well at all times if the pointer is **slanted**, to point towards the pole of the celestial sphere (go to http://www.phy6.org/stargaze/Sdial1.htm for an explanation —

but be warned, it is a bit complicated!). The angle between it and the base then equals the **geographic latitude** of the user.

5.1 A Paper Sundial

Ornamental sundials are often found in parks and gardens, with the pointer widened into a triangular fin, which must point northwards. A sundial of this type can be constructed from folded cardboard or stiff paper, this is shown at http://www.phy6.org/stargaze/Sdial2.htm to see the basic design used around latitude 38 North of the equator, go to http://www.phy6.org/stargaze/Sdial2S.htm for a corresponding one in the southern hemisphere.

Either can be printed and then photo-copied onto suitable sheets of stiff paper or cardboard [You may want to use the "option" menu to reduce size to 90% before printing–but make sure to return the setting to 100% afterwards!]. It is meant to be used at a latitude of 38 degrees and should work adequately in most of the continental US.

Instructions

1. Cut the paper along the marked line: one half will serve as **base**, the other will be used to construct the **gnomon**.
2. In the gnomon part, cut away the two marked corners.
3. Fold the sheet in its **middle**, in a way that the two secondary printed lines (leading to the cut-off corners) remain visible. The line of the fold is the gnomon.
   - **Note**: In stiff paper, straight folds are helped by first scoring the paper, by drawing a line along them with a black ballpoint, guided by a ruler and pressed down hard.
4. With the page folded in its middle, **cut out along the curved line**, cutting a double thickness of paper in one cut. The cut begins near the top of the gnomon-fold and ends on the secondary line. **Do not cut along the secondary line**. No pieces come off.
5. Score the other two secondary lines, then **fold** the gnomon sheet along them. The fold is **opposite** to that of the fold in the middle. These two folds should form 90-degree angles, so that the two pieces with the corners **not** cut in step 2 can be placed flat on the table, and the triangular gnomon rises above them.
6. In cut (4), the fin of the gnomon was separated from two pieces with curved outlines. **Fold** those pieces so that they, too, are flat with the table. One goes above the other, and the slots they form near the secondary lines create a place for the fin to fit into.
7. You are almost done. Take the **base** sheet, and note the **apex** where the hour-lines all meet (that is where the bottom corner of the fin will go). Carefully cut the sheet from this point along its middle line, up to the small cross-line marked on it. **Do not** cut any further!
8. **Slide the fin** into the cut you made, so that all horizontal parts of the first sheet are **below** the base sheet; only the fin sticks out. Its bottom corner should be at the apex.
   - **The sundial is now ready**, but you might use tape on the bottom of the base-sheet to hold the two pieces together firmly. For further stability, and to prevent the sundial from being blown away, you may attach its base with thumbtacks to a section of a **wooden board** or a piece of plywood.
9. Finally, **orient the fin** to point north. You may use a magnetic compass; before pocket watches were available, folding pocket sundials were used in Europe, with small magnetic compasses embedded in their bases. If clear sunlight is available, the shadow of the tip of the fin now tells the time.

If you want to make a sundial of more durable materials, draw the pre-noon hour lines at the angles to the fin (given in degrees) given below. These lines are meant for a latitude of 38 degrees; if your latitude is markedly different, see note at the end.

Table 5.1:

6: 90 degrees	7: 66.5 degrees	8: 46.8 degrees
9: 31.6 degrees	10: 19.6 degrees	11: 9.4 degrees

Accuracy

The sundial will obviously be one hour off during daylight saving time in the summer, when clocks are reset.

In addition, "clock time" (or "standard time") will differ from sundial time, because it is usually kept uniform across "time zones;" each time zone differs from its neighbors by one full hour (more in China and Alaska). In each such zone, sundial time matches clock time at only one geographical longitude: elsewhere a correction must be added, proportional to the difference in longitude from the locations where sundial time is exact.

> (Up to the second half of the 19th century, local time and sundial time were generally the same, and each city kept its own local time, as is still the case in Saudi Arabia. In the US standard time was introduced by the railroads, to help set up uniform timetables across the nation.)

Finally, a small periodic variation exists ("equation of time") amounting at most to about 15 minutes and contributed by two factors. First, the Earth's motion around the sun is an ellipse, not a circle, with slightly variable speed in accordance with Kepler's 2nd law (see http://www.phy6.org/stargaze/Skepl2A.htm as well as the section preceding that page). Secondly, the ecliptic (http://www.phy6.org/stargaze/Secliptc.htm) is inclined by 23.5 degrees to the equator, which means the projection of the Sun's apparent motion on it (which determines solar time) is slowed down near the crossing points of the two.

Note on Latitude

The angles listed above are intended for a latitude of 38 degrees. If your latitude is L, $\sqrt{}$ denotes "square root of" and

K $= \cot L^2 = \frac{\cos L^2}{\sin L^2}$

then the angle between the fin and the line corresponding to the hour N+6 (N going from 0 to 6) satisfies

$\sin A = \frac{\cos 15N}{\sqrt{1+K\sin 215N}}$

Here 15N (=15 times N) is an angle in degrees, ranging from 0 to 90, and of course, the afternoon angles are mirror reflections of the morning ones. If your calculator has a button ($\sin -1$), if you enter ($\sin A$) and press it, you will get the angle A. For an explanation of sines and cosines, look up the math refresher. And don't forget to adjust the angle of your fin to L, too!

And by the way

The sundial described here, with a gnomon pointing to the celestial pole, is a relatively recent invention, probably of the last 1000 years. Yet sundials were used long before, often with unequal hours at different times of the day. **The Bible** — 2nd book of Kings, chapter 20, verses 9-11 (also Isaiah, ch. 38, v. 8) tells of an "accidental" sundial, in which the number of steps covered by the Sun's shadow on a staircase was used to measure the passage of time.

5.2 Exploring Further

The "Sundial Bridge," with a unique design which may well make it the largest sundial anywhere, opened July 4, 2004, in Turtle Bay Park (http://www.turtlebay.org/sundialbridge) in Redding, California, at the foot of Mt. Shasta. Designed by the innovative Spanish architect Santiago Calatrava, it resembles his stunning 1992 bridge (http://www.galinsky.com/buildings/alamillo/) erected in Seville, Spain. It is a pedestrian bridge, connecting two parts of Turtle Bay Park, and it also operates as a sundial, using plaques set in a semicircular upper plaza.

For a more detailed article about this bridge, see Sundial Bridge at Turtle Bay on wikipedia: http://en.wikipedia.org/wiki/Sundial_Bridge_at_Turtle_Bay.

Before the days of affordable wristwatches, people often carried a folding sundial in their pocket ("poke" below), with a small magnetic compass embedded, to show the north direction. In "**As You Like It**" by **William Shakespeare** (act 2, scene 7) one of the characters tells of meeting in the forest a fool (witty court entertainer) carrying such a "dial":

> "Good morrow, fool," quoth I. "No, sir," quoth he,
>
> "Call me not fool till heaven sent me fortune:"
>
> And then he drew a dial from his poke,
>
> And, looking on it with lack-lustre eye
>
> Says very wisely, "It is ten o'clock:
>
> Thus we may see how the world wags:
>
> 'T is but an hour ago since it was nine;
>
> And after one hour more 't will be eleven;
>
> (and continues)

You may also be interested to know that a North American Sundial Society (NASS) exists, with its home page at http://sundials.org.. The British Sundial Society also has its sundial page at http://www.sundials.co.uk/.

A sundial was included as part of the Mars lander mission and is shown in "Astronomy Picture of the Day" for April 28, 1999. It has a thick vertical gnomon, so that its readings may need some extra corrections.

For those with serious interest in history (and access to a good library): "**The Material Culture of Astronomy in Daily Life: Sundials, Science and Social Change**" by Sara Schechner (History of Sci. Dept., Harvard) Journal for the History of Astronomy Vol. 32, part 3, August 2001, p. 189-222, with many illustrations.

From the 1.1.2000 book catalog of Willman-Bell in Richmond, Virginia (www.willbell.com):

- **Easy-to-make Wooden Sundials**, by Stoneman, 38 pp., $4.95
- **Sundials: History, Theory and Practice by Rohr**, 230 pp, $12.95.
- **Sundials: Their Theory and Construction by Waugh**, 19 chapt., $8.95

Image Sources

(1) Alex Zaliznyak, David Stern. *A Sundial*. CC-BY-SA 3.0.

Chapter 6

Seasons of the Year

The first section (See the chapter “Stargazers and Sunwatchers” in the *From Stargazers to Starships FlexBook* on www.ck12.org) described the **observed motion** of the Sun across the sky, in different seasons of the year. **This** section tries **to explain** what is seen.

If the Earth’s axis were **perpendicular** to the ecliptic, as in the drawing below, the Sun’s position in the sky would be halfway between the celestial poles, and its daily path, seen from any point on Earth, would stay exactly the same, day after day.

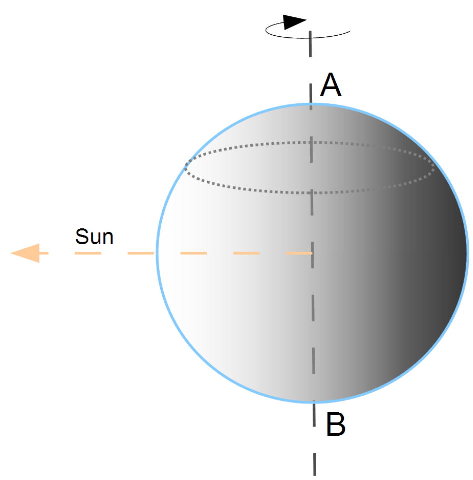

Figure 6.1: An Earth rotating on an axis perpendicular to the ecliptic.

Each point on Earth would be carried around the axis AB once a day. On the equator (point C) the sun would always rise until it was overhead, then again descend to the horizon. At the poles (A and B) it would always graze the horizon and never get away from it. Except at the pole, every point would be in the shadow half the time, when on the right of the line AB, and would experience night; the other half it would be in the sunlight, experiencing day. Because the motion is symmetric with respect to the line AB, day and night anywhere on Earth are always equal.

Actually, the axis of rotation makes an angle of about 23. 5 degrees with the direction perpendicular to the ecliptic. That makes life **a lot more interesting**.

6.1 Equinox and Solstice

In particular, the angle between the Earth's axis and the Earth-Sun line changes throughout the year. Twice a year, at the spring and fall **equinox** (around March 21 and September 22–the exact date may vary a bit) the two directions are **perpendicular**.

Twice a year, the angle is as big as it can get, at the summer and winter **solstices**, when it reaches 23.5 degrees. In the summer solstice (around June 21) the north pole is inclined **towards** the Sun, in the winter solstice (around December 21) it faces **away** from it.

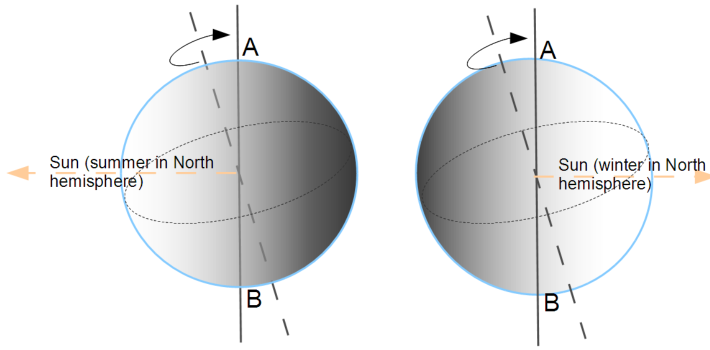

Figure 6.2: Winter and summer in the Northern hemisphere.

Let us look at the summer first, with the Sun on the left.

6.2 Summer and Winter

The boundary AB between sunlight and shadow — between day and night — is **always** perpendicular to the Earth-Sun line, as it was in the example shown at the beginning.

But because of the tilted axis, as each point on Earth is carried on its daily trip around the rotating Earth, the part of the trip spent in daylight (unshaded part of the drawing) and in the shadow (shaded) are usually not equal. North of the equator, day is longer than night, and when we get close enough to the north pole, there is no night at all. The Sun is then **always above the horizon and it just makes a 360-degree circuit around it. That part of Earth enjoys summer**.

A mirror-image situation exists south of the equator. Nights are longer than days, and the further one gets from the equator, the larger is the imbalance–until one gets so close to the pole that **the sun never rises**. That is the famous polar night, with 24 hours of darkness each day. In that half of the Earth, it is **winter** time.

Half a year later, the Earth is on the other side of the Sun, that is, the Sun's position in the above drawing should be on the **right**, and the shaded part of the Earth should now be on the **left** (light and dark portions in the drawing switch places). The Earth's axis however has not moved, it is still pointed to the same patch of sky, near the star Polaris. Now the south pole is bathed in constant sunshine and the north one is dark. Summer and winter have switched hemispheres.

A big difference between summer and winter is thus the length of the days: note that on the equator that length does not change, and hence Spring, Summer, Fall, and Winter do not exist there (depending on weather patterns, however, there may exist a "wet season" and a "dry season"). In addition (as the drawing makes clear), the Sun's rays hit the summer hemisphere more vertically than the winter one. That, too, helps heat the ground, as explained further in the chapter "The Angle of the Sun's Rays" in the *From*

Stargazers to Starships FlexBook on www.ck12.org.

At equinox, the situation is as in the first drawing, and night and day are equal (that is where the word "equinox" comes from).

6.3 Some Interesting Facts

If June 21 is the day when we receive the most sunshine, why is it regarded as the **beginning** of summer and not its peak? And similarly, why is December 21, the day of least sunshine, the **beginning** of winter and not mid-winter day?

Blame the oceans, which heat up and cool down only slowly. By June 21 they are still cool from the winter time, and that delays the peak heat by about a month and a half. Similarly, in December the water still holds warmth from the summer, and the coldest days are still (on the average–not always!) a month and a half ahead.

And what about our **distance from the Sun**? It, too, varies, because the Earth's orbit around the Sun isn't an exact circle. We are closest to the Sun — would you believe it? — in the cold wintertime, around January 3-5. This may have an interesting implication for the origin of **ice ages**, as will be explained later. It also ties to an interesting story of the unusually bright Moon of December 22, 1999, see http://www.phy6.org/stargaze/Imoon3.htm.

Image Sources

(1) David Stern, Alex Zaliznyak. *Seasons 1*. CC-BY-SA 3.0.

(2) Alex Zaliznyak, David Stern. *Seasons 2*. CC-BY-SA 3.0.

Chapter 7

The Angle of the Sun's Rays

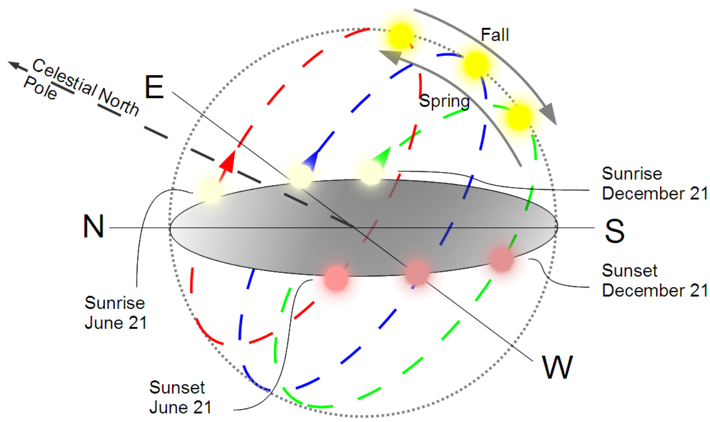

Figure 7.1: . Note how much higher the sun is during summer (green circle)

In the US and in other mid-latitude countries north of the equator (e.g. those of Europe), the sun's daily trip (as it appears to us) is an arc across the southern sky. (Of course, it's really the Earth that does the moving.) The **sun's greatest height** above the horizon occurs at noon, and how high the sun then gets depends on the season of the year–it is highest in mid-summer, lowest in mid-winter.

Boy scouts used to be taught (perhaps still are) that someone lost in the woods can often tell the north direction by checking on which side of tree-trunks lichens grew best. Lichens avoid direct sunlight, and with the sun's path curving across the southern sky, the north side of a tree-trunk is the one most shaded.

For a similar reason–but to collect sunlight rather than avoid it–solar collectors for heating water or generating electricity always face south. In addition, they are invariably tilted at an angle around 45°, to make sure that the arrival of the sun's rays is as close to perpendicular as possible. The collector is then exposed to the highest concentration of sunlight: as the drawing shows, if the sun is 45 degrees above the horizon, a collector 0.7 meters wide perpendicular to its rays intercepts as much sunlight as a 1-meter collector flat on the ground. It therefore heats its water faster and reaches a higher temperature. French wine producers, too, have for centuries preferred southward-facing hillsides, on which ripening grapes get the most sunlight.

The same also holds for the Earth. The rays of the summer sun, high in the sky, arrive at a steep angle and heat the land much more than those of the winter sun, which hit at a shallow angle. Although the length of the day is an important factor in explaining why summers are hot and winter cold, the angle of

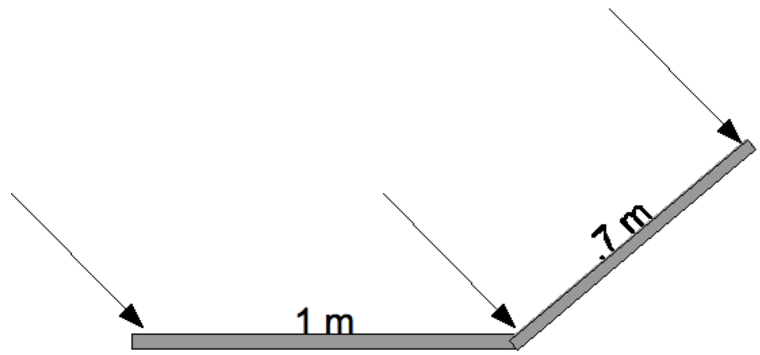

Figure 7.2: The Angle of the Sun's Rays.

sunlight is probably more important. In the arctic summer, even though the sun shines 24 hours a day, it produces only moderate warmth, because it skims around the horizon and its light arrives at a low angle.

The apparent motion of the sun can be important in designing a building, in particular in the placing of windows, which trap the sun's heat. In a hot sunny climate such as that of Texas or Arizona, it is best to have the largest windows face north, avoiding the sun. The south-facing walls, on the other hand, should be well insulated and their windows should be small, allowing cross-ventilation when needed but not admitting much sunlight (wooden shutters on the outside of the windows also help). In Canada the opposite directions might be chosen, to trap as much heat as possible from the winter sun.

Overhangs above south-facing windows also help. In summer, with the noontime Sun high in the sky, such an overhang casts a shadow on the window and keeps the house cool. In the winter, however, when the Sun stays close to the horizon, the overhang allows it to shine through the window and warm the rooms inside.

Image Sources

(1) Alex Zaliznyak, David Stern. *Path of the Sun*. CC-BY-SA 3.0.

(2) David Stern, Alex Zaliznyak. *Angle of the Sun's Rays*. CC-BY-SA 3.0.

Chapter 8

The Moon: the Distant View

The Moon's the North Wind's cookie
He bites it, day by day
Until there's but a rim of scraps
That crumble all away.
The South Wind is the baker
He kneads clouds in his den,
And bakes a crisp new moon that ...
greedy.... North.... Wind ....eats....again!

—"*What the Little Girl Said*"
Vachel (Nicholas) Lindsay, 1879-1931.

8.1 The Month

The monthly cycle of the moon (we won't capitalize the word here) must have mystified early humans — "waxing" from thin crescent ("new moon") to half-moon, then to a "gibbous" moon and a full one, and afterwards "waning" to a crescent again. That cycle, lasting about 29.5 days, gave us the word "month" — related to "moon," as is "Monday."

The civil year, January to December, no longer ties its months to the moon, but some traditions still do and their terms for "month" reflect the connection — in Arabic, "shahr," in biblical Hebrew "yerach" and also "chodesh" from "new," since it was reckoned from one new moon to the next. Jericho (pronounced Yericho), one of the oldest cities on Earth, took its name from "yerach," and of course, legends tell of many moon-gods and goddesses, e.g. Artemis and Diana.

Early astronomers understood the different shapes of the moon, noting that each was linked to a certain relative position between moon and Sun: for instance, full moon always occurred when moon and Sun were at opposite ends of the sky. All this suggested that the moon was a sphere, illuminated by the Sun.

The moon's path across the sky was found to be close to the ecliptic, inclined to it by about 5 degrees. Eclipses of the Sun always occurred when moon and Sun were due to occupy the same spot in the sky, suggesting that the moon was nearer to us and obscured the Sun. Eclipses of the moon, similarly, always

Figure 8.1: A Full Moon.

occurred at full moon, with the two on opposite sides of the Earth, and could be explained by the shadow of the Earth falling on the moon.

Lunar eclipses allowed the Greek astronomer Aristarchus, around 220 BC, to estimate the **distance to the moon** (See the chapter "Estimating the Distance to the Moon" in the *From Stargazers to Starships FlexBook* on www.ck12.org). If the moon and the Sun followed **exactly the same path** across the sky, eclipses of both kinds would happen each month. Actually they are relatively rare, because the 5-degree angle between the paths only allows eclipses when Sun and moon are near one of the points where the paths intersect.

The cycle from each new moon to next one takes **29.5 days**, but the actual orbital period of the moon is only **27.3217 days**. That is the time it takes the moon to return to (approximately) the same position among the stars.

Why the difference? Suppose we start counting from the moment when the moon in its motion across the sky is just overtaking the Sun; we will call this the "new moon," even though the thin crescent of the moon will only be visible some time later, and only shortly after sunset. Wait 27.3217 days: the moon has returned to approximately the same place in the sky, but the Sun has meanwhile moved away, on its annual journey around the heavens. It takes the moon about 2 more days to catch up with the Sun, to the position of the next "new moon," which is why times of the new moon are separated by 29.5 days.

8.2 The Face of the Moon

The visible face of the moon has light and dark patches, which people interpreted in different ways, depending on their culture. Europeans see a face and talk of "the man in the moon" while children in China and Thailand recognize "the rabbit in the moon." All agree, however, that the moon does not change, that it **always presents the same face to Earth.**

Does that mean the moon doesn't rotate? No, it **does** rotate — one rotation for each revolution around Earth! The accompanying drawings, covering half an orbit, should make this clear. In them we look at the moon's orbit from high above the north pole, and imagine a clock dial around the moon, and a feature on it, marked by an arrow, which initially (bottom position in each drawing) points at 12 o'clock. In the

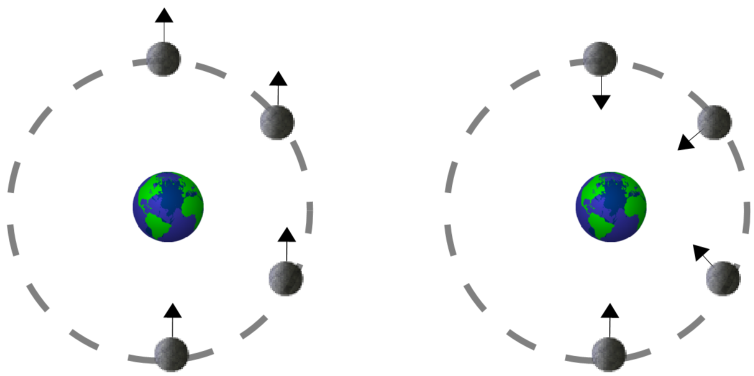

Figure 8.2: Half of the moon's orbit.

right drawing the marked feature continues to point at Earth, and as the moon goes around the Earth, it points to the hours 10, 8 and 6 on the clock dial. As the moon goes through half a revolution, it also undergoes half a rotation. If the moon **did not** rotate, the situation would be as in the **left drawing**. The arrow would continue to point in the 12-oclock direction, and after half an orbit, people on Earth would be able to see the other side of the moon. This does **not** happen.

We need to go aboard a spaceship and fly halfway around the Moon before we get a view of its other side — as did the Apollo astronauts who took the picture below.

8.3 The Gravity Gradient

This strange rotation of the moon is maintained because the moon is slightly elongated along the axis which points towards earth. To understand the effect we look at the motion of a body with a much more pronounced elongation — an artificial satellite with the shape of a symmetric dumbbell (see **Figure** 8.4).

Itgg can be shown that if the forces on the dumbbell (or indeed on a satellite of any shape) are unbalanced, it rotates around its **center of gravity**. That point will be defined here http://www.phy6.org/stargaze/Srocket.htm, but in a symmetric dumbbell with two equal masses marked A and B, the center of gravity is right in the middle between them.

Both masses A and B are attracted to the Earth, and if the attracting forces were equal, their tendencies to rotate the satellite ("rotation moments" or "torques") are equal and cancel each other, so that no rotation occurs. If however **A starts closer** to the center of Earth, the force on it is **just a little stronger**. Therefore the satellite will rotate until A is as close to Earth as it can be, which is a possible position of equilibrium. Of course, it may then overshoot its equilibrium position, and end up swinging back-and-forth like a pendulum, only slowly (like a pendulum) losing energy and settling down. The elongated moon acts like a dumbbell too.

The rotating force which lines up the moon or an orbiting dumbbell is the **difference** between the pull on A and on B. It depends not on **how strongly** gravity pulls these masses, but on how rapidly the pull of gravity **changes with distance** — on the "gravity gradient." Near Earth that is a gentle force, though still strong enough to line up elongated satellites. Among those was Triad, (http://www.phy6.

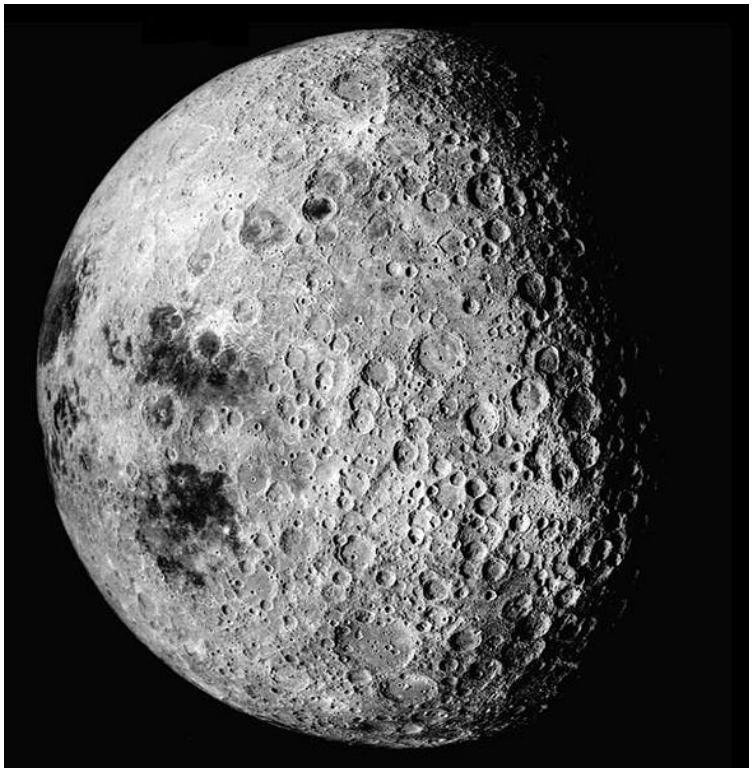

Figure 8.3: The far side of the moon.

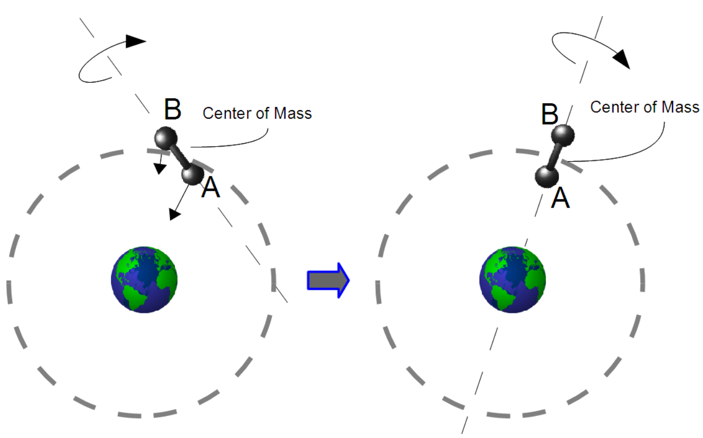

Figure 8.4: Illustration of the gravity gradient concept: the difference in the force of gravity experienced by parts of the satellite will cause it to line its axis of rotation perpendicular to its orbit around the earth.

org/Education/wtriad.html) deliberately shaped like a long dumbbell with an additional payload in the middle, the first satellite to map the electrical currents associated with the polar aurora.

Near a black hole or pulsar, though, the gravity-gradient force can be fierce enough to rip a spacecraft apart.

Actually, the long axis of the Moon does not always point **exactly** to the center of the Earth, but swings back and forth around that direction, a motion known as **libration**. Most of this is caused because the Moon rotates around its axis with a fixed period, while its motion around its orbit slows down far from Earth and speeds up close to it. This speeding up and slowing down is the result of Kepler's 2nd law, discussed in section 12, and is a rather small effect, since the moon's orbit is very close to circular.

Because of libration, even though at any time only half the Moon is visible, over time 59% can be seen, since it lets astronomers look at the Moon from slightly different viewing directions. Librations are a rather specialized subject — but if you want to know more about it, go to http://www.phy6.org/stargaze/Smoon4.htm

8.4 Earthshine

At times when only a narrow crescent of the Moon is seen (e.g. a "new moon"), one can also see the rest of the Moon faintly outlined. The Sun now shines on almost all of the side of the moon turned **away** from Earth (those calling that "the dark side of the moon" are quite wrong!) and therefore it also illuminates most of the side of the Earth **facing** the moon. If you were standing on the moon at that time, a "**full Earth**" would shine brightly in your sky, and the faint "earthshine" of the darker part of the moon is just the reflection of some of that bright earthlight.

Earthshine is of interest to scientists, because its brightness is contributed by all the factors which turn back sunlight before it manages to heat the Earth–light reflected from the ground and from clouds, and light scattered back by dust and small particles ("aerosols") in the atmosphere. In a time when atmospheric scientists are trying to assess heating of the Earth by the greenhouse effect (http://www.phy6.org/stargaze/Sun1lite.htm), earthshine measures a process which works in the opposite direction, reducing the heat our planet receives.

The fraction of light reflected is hard to estimate theoretically, but earthshine allows it to be measured. According to recent reports ("The Darkening Earth," Scientific American August 2004, p. 16), this fraction has been growing, reducing the amount of sunlight received by Earth and **canceling about 1/3 of the greenhouse heating**.

Image Sources

(1) NASA. *Far Side of the Moon*. Public Domain.

(2) Luc Viatour. *Full Moon*. GNU Free Documentation License.

(3) Alex Zaliznyak, David Stern. *Gravity Gradient*. CC-BY-SA 3.0.

(4) David Stern, Alex Zaliznyak. *Moon Orbit*. CC-BY-SA 3.0.

Chapter 9

The Moon: a Closer Look

9.1 The View Through the Telescope

When Galileo (http://www.phy6.org/stargaze/Ssolsys.htm#q1C) became the first human to view the Moon through a telescope, our understanding of the Moon changed forever. No longer a mysterious object in the sky, but a sister-world full of ring-shaped mountains and other formations!

Giovanni Riccioli in 1651 named the more prominent features after famous astronomers, while the large dark and smooth areas he called "seas" or "maria" (singular "mare," mah-reh). Some of the names he used for the Moon's crater are of persons discussed in "Stargazers"—**Tycho** (distinguished by bright streaks that radiate from it), **Ptolemy** ("Ptolemaeus"), **Copernicus**, **Kepler**, **Aristarchus**, **Hipparchus**, **Erathosthenes**; **Meton** and **Pythagoras** are on the edge, near the northern pole.

Late-comers who lived after the 17th century had to make do with left-overs: the craters **Newton** and **Cavendish** are at the southern edge of the visible disk, **Goddard** and **Lagrange** too are near the edge. Also, "**Galilaei**" is a small undistinguished crater (because of Galileo's banishment?). However, since the Russians were the first to observe the rear side of the Moon, a prominent crater there bears the name of **Tsiolkovsky**, who at the end of the 19th century promoted the idea of spaceflight.

9.2 The Craters

What had created those strange round "craters?" ("Krater" is Greek for a bowl or wide-mouthed goblet.) They reminded some observers of volcanic craters on Earth, or better, of the large "calderas" (cauldrons) formed by the internal collapse of volcanos, e.g. Crater Lake (http://www.crater.lake.national-park.com/) in Oregon. Others suggested that they were formed by the impact of large meteorites, but this was countered by the argument that most meteorites probably arrived at a slanting angle, and were expected to leave not a round ring but an elongated gouge.

We now know that the impact explanation was right. The craters are round because at the enormous velocities with which meteorites arrive, the impact resembles a local explosion, and the signature of the impact is determined by the energy released rather than by the momentum transmitted.

Part of the evidence has come from the nicely rounded impact remnants found on Earth, e.g. Meteor Crater (Canyon Diablo) in Arizona and Manicouagan lake in Canada, in northern Quebec (see picture), which is about 100 km (60 miles) wide and 214 million years old. Note that rather than having a pit in its center, the **Manicougan** lake has a round island. After the impact, the land rose again to the level of its surroundings, pushed by fluid pressure of the material below it, which acts like a viscous fluid and tries to establish equilibrium between the different loads which it supports. (For another picture of Lake Manicouagan, and more about it, click here: http://antwrp.gsfc.nasa.gov/apod/ap001213.html)

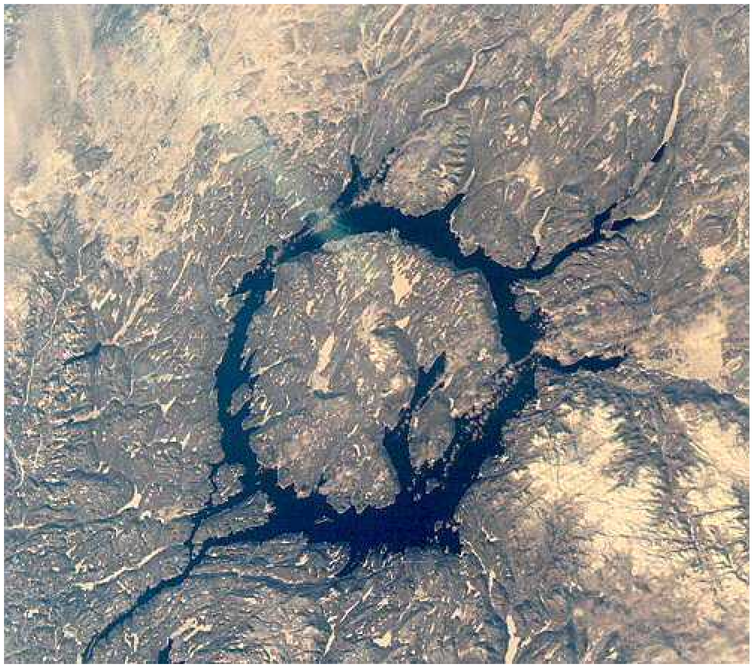

Figure 9.1: Manicouagan lake in Canada.

Other solid bodies of the solar system also diplay round craters. On the large ice-covered moons of Jupiter, the return to equilibrium is much more pronounced, because ice sags and flows much more readily than rock. Those moons display "palimpsest" (named so after scrolls of parchment that were 'recycled' by having their original text scratched off) craters which are merely surface markings, because as time passed, the walls which originally existed sagged back onto the flat surface.

9.3 The Airless Moon

In the centuries after Galileo's discoveries, the Moon was extensively studied by astronomers using telescopes. One thing soon became clear: it had no atmosphere. When a star was eclipsed by the Moon, it vanished suddenly and its light showed no refraction or absorption by an atmosphere.

Why? By the laws of motion, the Moon orbits not the center of the Earth, but the center of gravity of the Earth and Moon (this is discusses at http://www.phy6.org/stargaze/Skepl1st.htm#q201, and the center of gravity is defined in http://www.phy6.org/stargaze/Srocket.htm). The location of that point allows astronomers to deduce the mass of the Moon, and from that, the pull of the Moon's gravity. At the surface of the Moon, it turned out, gravity is only 1/6 as strong as at the surface of the Earth.

Gravity is important for the retention of an atmosphere. It holds an atmosphere down, while **heat** is what can make it escape.

Heat is related to atomic or molecular motion. In solids and liquids, atoms or molecules can vibrate around their average position. The higher the object's temperature, the more vigorous the motion, until the material boils or evaporates, at which point its particles shake loose altogether. In a gas atoms and molecules fly around randomly, colliding constantly (if the gas is as dense as it is in the atmosphere), and their collisions lead to a very good explanation ("the kinetic theory of gases") of the observed properties of a gas.

The average velocity of a gas molecule depends on the temperature of the gas, and at room temperature it is comparable to that of the speeding bullet, quite below the "escape velocity" needed for escaping Earth's gravity. However, that is just an **average**: actual velocities are expected to be distributed around that average, following the "**Maxwellian distribution**" first derived by James Clerk Maxwell, whom we meet again in the discovery of the three color theory of light (http://www.phy6.org/stargaze/Sun4spec.htm#q1C) and the prediction of electromagnetic waves (http://www.phy6.org/stargaze/Sun5wave.htm#q38A). According to that distribution, a few molecules always move fast enough to escape, and if they happen to be near the top of the atmosphere, moving upwards and and avoiding any further collisions, such molecules would be lost.

For Earth, their number is too small to matter, but with the Moon, having only 1/6 of the surface gravity, it can be shown that any atmosphere would be lost within geological time. The planet Mercury, only slightly larger, also lacks any atmosphere, while Mars, with 1/3 the Earth's surface gravity, only retains a very thin atmosphere.

Water evaporates easily and once in gas form, is quickly lost by the same process. That suggested the "maria" could not possibly be oceans, though their name remained. They actually turned out to be basaltic flows, hardened lava which long ago flowed out of fissures on the Moon; no present-day volcanism on the Moon has been reliably identified. The vast majority of craters probably date back to the early days of the solar system, because the lava of the maria has very few craters on it, suggesting it flooded and obliterated older ones.

The picture of a dry Moon was reinforced by Moon rocks brought back by US astronauts. Earth rocks may contain water bound chemically ("water of hydration"), but not these. Water, of course, would be essential to any human outpost on the Moon. Yet small amounts of water may still exist, brought by comets which occasionally hit the Moon. All this water is sure to evaporate in the heat of the collision, but some of it may re-condense in deep craters near the Moon's pole, which are permanently in the shade and therefore extremely cold. Observations by the "Clementine" spacecraft suggest that one such crater may indeed contain a layer of ice.

9.4 In the Space Age

From the beginning of spaceflight, the Moon was a prime target, but this chapter in space exploration is too long to be covered here in any detail. The first spacecraft to reach the Moon were Luna 1, 2 and 3 of the Soviet Union, in 1959. Of these, Luna 3 rounded the Moon, took photographs of the far side which is not seen from Earth, and later scanned and transmitted those images (see **Figure** 9.2); unfortunately, their quality was poor. In the decade that followed, 19 other Soviet missions were aimed at the Moon.

In 1970 a Soviet spacecraft landed and returned a rock sample, and later that year a remotely controlled "Lunokhod" (literally: moon-walker, shown in **Figure** 9.3) vehicle was landed, exploring its surroundings for nearly a year. Other sample returns and Lunokhods followed, the series ending in 1976. However, failures marked tests of a large rocket developed for human Moon flights, ending any plans of manned lunar exploration by the Soviet Union.

Early attempts by the US to send unmanned spacecraft to the Moon (1958-64) either failed or returned scanty data. In July 1964, however, Ranger 7 returned clear TV pictures of its impact on the Moon, as did Rangers 8 and 9. Of the 7 "soft landers" in the "Surveyor" series (1966-8), 5 performed well and sent back data and pictures. In November 1969, after Apollo 12 landed 500 feet (160 meters) from the "Surveyor 3" lander, astronauts retrieved its camera and brought if back to Earth. In addition to the Surveyor project, 5 lunar orbiters photographed the Moon and helped produce accurate maps of its surface.

On May 25, 1961, about one month after Russia's Yuri Gagarin became the first human to orbit the globe, US president John F. Kennedy proposed to the US Congress "that this nation should commit itself to

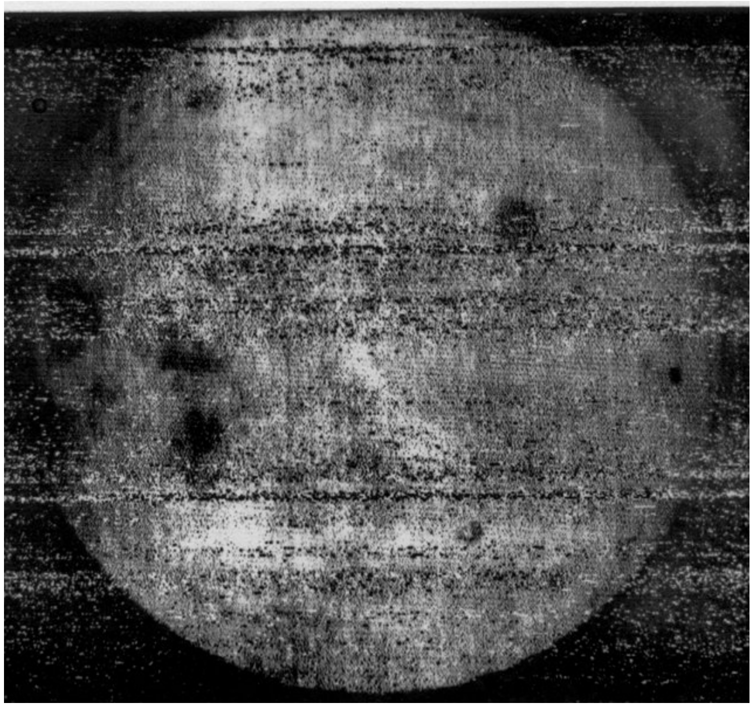

Figure 9.2: An Image from Luna 3.

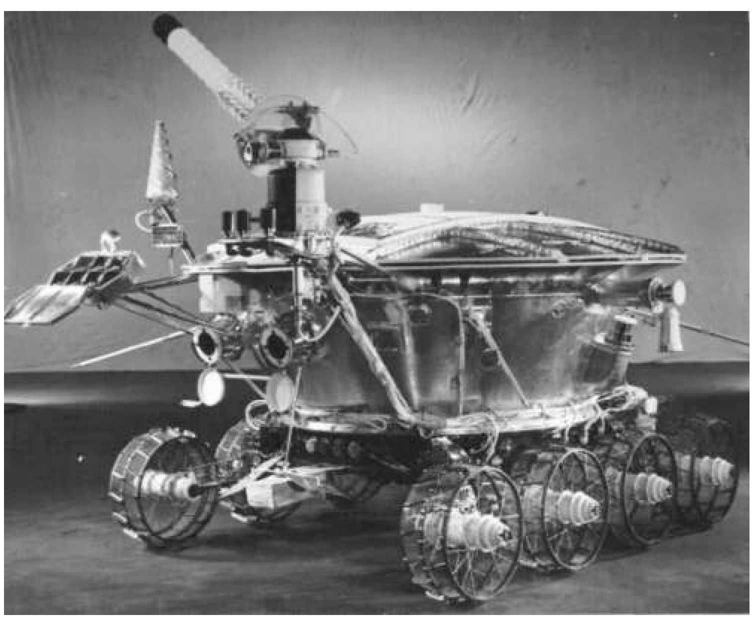

Figure 9.3: The Lunokhod Moon vehicle.

achieving the goal, before this decade is out, of landing a man on the Moon and returning him safely to Earth. "

The Apollo missions followed, with Apollo 8 rounding the Moon in 1968 and Apollo 11 finally landing there, on July 20, 1969 (pictured in **Figure** 9.4). Five other lunar landings followed, the last of them in December 1972. Only Apollo 13 failed to land, its crew members narrowly escaping with their lives after an explosion aboard their craft on the way to the Moon.

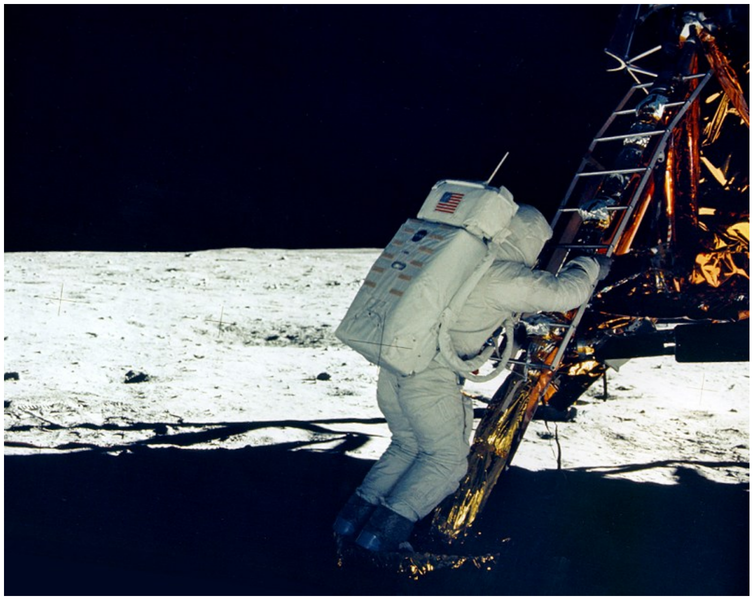

Figure 9.4: Apollo 11 Astronaut Buzz Aldrin, the Second Man on the Moon.

9.5 Achievements of "Project Apollo"

Among the activities of Apollo astronauts on the Moon were:

- Bringing back to Earth extensive samples of lunar rock and soil. The rocks turned out to be ancient, suggesting no significant change since the surface of the Moon formed, about 4.5 billion years ago. The "soil" (regolith) had probably been pulverized by impacts, but as the "Surveyor" missions showed, it was firm enough to provide support.
- Crews of Apollo 15, 16 and 17 explored the Moon aboard an electrically driven "moon buggy. "
- Extensive video pictures from the Moon were beamed to Earth–even one (by a remotely controlled camera) of the take-off from the Moon by the Apollo 17 crew. Also, the Earth and its "geocorona" of glowing hydrogen were photographed by a special camera using ultraviolet light.
- A seismometer was placed on the Moon, showing that the Moon was seismically much quieter than Earth.
- Metal foils were hung out (like a flag) to receive the solar wind. They were then returned to Earth where the composition of the ions caught in them was analyzed.
- Corner reflectors were placed on the Moon, so that laser beams reflected from them could accurately measure the distance.

9.6 Since Apollo

No humans have visited the Moon from 1972 until now, but some orbital missions have studied the Moon's magnetic field as well as X-ray and gamma-ray emissions, from which some variations of the surface composition could be inferred.

The Moon was found to have no global magnetic field like the Earth, but its surface was weakly magnetized in some patches. Molten rock can become permanently magnetized if it solidifies in the presence of an external magnetic field, suggesting that in some ancient era the Moon, like Earth now, had a molten metallic core in which electric currents generated a magnetic field. Somewhat similar observations were made near Mars in 1998-2000.

Some excitement was caused by indications from the Lunar Prospector (http://en.wikipedia.org/wiki/Lunar_Prospector) spacecraft, which suggested that ice may exist on the moon, inside a deep crater near the Moon's south pole. A possible explanation was that some time in the past (perhaps long ago) a comet had crashed into the Moon, and comets contain considerable amounts of water ice. The energy of the impact, turned into heat, would of course evaporate the ice. However, some of the water vapor would form a temporary atmosphere around the Moon, and might condense again to ice in very cold locations, like craters near the pole, which are permanently shaded from sunlight.

At the end of its mission, on July 31, 1999, Lunar Prospector was therefore steered to deliberately crash inside the crater. It was hoped the impact might create (briefly) a cloud of water vapor, which could be observed from Earth, but none was detected (http://science.nasa.gov/newhome/headlines/ast03sep99_1.htm).

There is little doubt that the future will see further lunar exploration, though a "lunar base" is probably far off. Astronomical and other observations can readily be made from Earth orbit, and providing life support on the Moon is not easy. Such a base will probably become attractive only after ways are developed for utilizing local lunar materials for construction and for fuel.

9.7 Exploring Further

Entire books about "Project Apollo" can be found of the web. Some good ones:

- *Apollo–Expeditions to the Moon*, Edited by Edgar M. Cortright. (http://www.hq.nasa.gov/office/pao/History/SP-350/cover.html)
- "Contact Light" by Kipp Teague. (http://www.retroweb.com/contact_light.html)
- *Where no Man has gone before* by the NASA History Office. (http://www.hq.nasa.gov/office/pao/History/SP-4214/cover.html)

A timeline of lunar landings with links to further details of the missions can be found here: http://nssdc.gsfc.nasa.gov/planetary/lunar/lunartimeline.html

Finally, here's a site about impact craters on earth: http://www.lpi.usra.edu/publications/slidesets/impacts.html

Image Sources

(1) Soviet Space Agency. *Luna 3*. Public Domain.

(2) NASA. *Manicougan Lake*. Public Domain.

(3) Soviet Space Agency. *Lunokhod 1*. Public Domain.

(4) NASA. *Apollo 11*. Public Domain.

Chapter 10

Latitude and Longitude

Any location on Earth is described by two numbers — its **latitude** and its **longitude**. If a pilot or a ship's captain wants to specify position on a map, these are the "coordinates" they would use.

Actually, these are two **angles**, measured in degrees, "minutes of arc" and "seconds of arc." These are denoted by symbols e.g. 35° 43' 9" means an angle of 35 degrees, 43 minutes and 9 seconds (do not confuse this with the notation (',") for feet and inches!). A degree contains 60 minutes of arc and a minute contains 60 seconds of arc — and you may omit the words "of arc" where the context makes it absolutely clear that these are **not** units of time.

Calculations often represent angles by small letters of the Greek alphabet, and that way latitude will be represented by λ (lambda, Greek L), and longitude by ϕ (phi, Greek F). Here is how they are defined.

> PLEASE NOTE: Charts used in ocean navigation often use the OPPOSITE notation — λ for LONGITUDE and ϕ for LATITUDE. The convention followed here resembles the one used by mathematicians in 3 dimensions for spherical polar coordinates (http://www.phy6.org/stargaze/Scelcoor.htm#q65.

10.1 Latitude

Imagine the Earth was a **transparent sphere** (actually the shape is slightly oval; because of the Earth's rotation, its equator bulges out a little). Through the transparent Earth we can see its equatorial plane, and its middle the point is O, the center of the Earth.

To specify the latitude of some point P on the surface, draw the radius OP to that point. Then the **elevation angle** of that point **above the equator** is its latitude λ — northern latitude if north of the equator, southern (or negative) latitude if south of it.

> How can one define the angle between a line and a plane, you may well ask? After all, angles are usually measured between two **lines**!
>
> Good question. We must use the angle which completes it to 90 degrees, the one between the given line and one **perpendicular** to the plane. Here that would be the angle (90°-λ) between OP and the Earth's axis, known as the **co-latitude** of P.

On a globe of the Earth, lines of latitude are circles of different size. The longest is the **equator**, whose

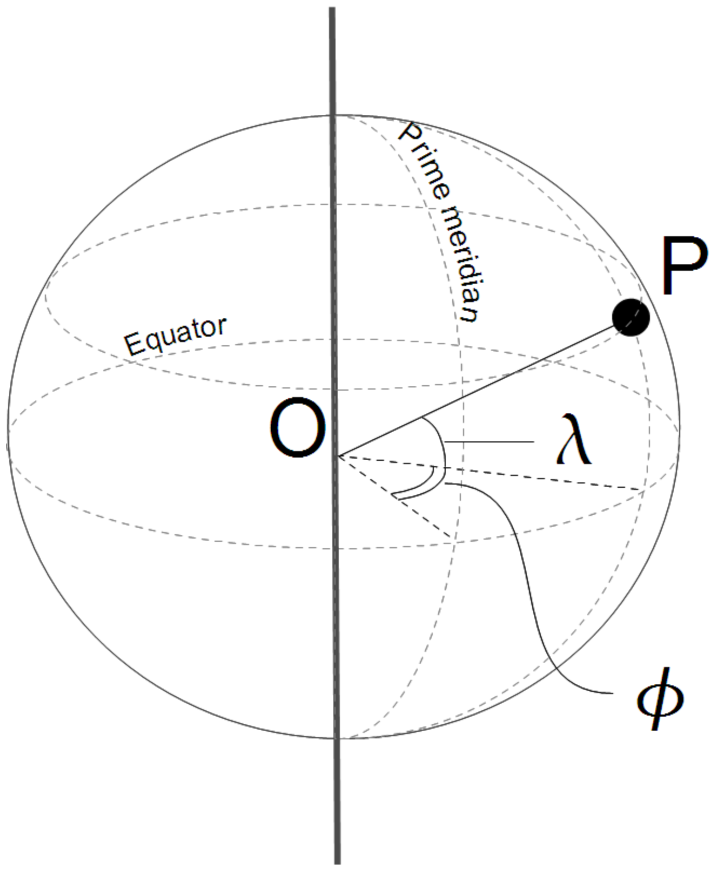

Figure 10.1: An illustration of longitude and latitude.

latitude is zero, while at the poles–at latitudes 90° north and 90° south (or -90°) the circles shrink to a point.

10.2 Longitude

On the globe, lines of constant longitude ("meridians") extend from **pole to pole**, like the segment boundaries on a peeled orange.

Every meridian must cross the equator. Since the equator is a circle, we can divide it — like any circle — into 360 degrees, and the **longitude ϕ of a point** is then the marked value of that division where its meridian meets the equator.

What that value is depends of course on where we begin to count — on where **zero longitude** is. For historical reasons, the meridian passing the old Royal Astronomical Observatory in Greenwich, England, is the one chosen as zero longitude. Located at the eastern edge of London, the British capital, the observatory is now a public museum and a brass band stretching across its yard marks the "prime meridian." Tourists often get photographed as they straddle it–one foot in the eastern hemisphere of the Earth, the other in the western hemisphere.

A lines of longitude is also called a **meridian**, derived from the Latin **meri**, a variation of "medius" which denotes "middle," and **diem**, meaning "day." The word once meant "noon," and times of the day before noon were known as "ante meridian," while times after it were "post meridian." Today's abbreviations **a.m. and p.m.** come from these terms, and the Sun at noon was said to be "passing meridian." All points on the same line of longitude experienced noon (and any other hour) at the same time and were therefore said to be on the same "meridian line," which became "meridian" for short.

10.3 About Time: Local and Universal

Two important concepts, related to latitude and (especially) longitude are **Local time (LT)** and **Universal time (UT).**

Local time is actually a measure of the position of the Sun relative to a locality. At 12 noon local time the Sun passes to the south and is furthest from the horizon (northern hemisphere). Somewhere around 6 am it rises, and around 6 pm it sets. Local time is what you and I use to regulate our lives locally, our work times, meals and sleep-times.

But suppose we wanted to time an astronomical event — e.g. the time when the 1987 supernova was first detected. For that we need a single agreed-on clock, marking time world-wide, not tied to our locality. That is **universal time** (UT), which can be defined (with some slight imprecision, no concern here) as the local time in Greenwich, England, at the zero meridian.

10.4 Local Time (LT) and Time Zones

Longitudes are measured from zero to 180° east and 180° west (or -180°), and both 180-degree longitudes share the same line, in the middle of the Pacific Ocean.

As the Earth rotates around its axis, at any moment one line of longitude — "**the noon meridian**" — faces the Sun, and at that moment, it will be **noon** everywhere on it. After 24 hours the Earth has undergone a full rotation with respect to the Sun, and the same meridian again faces noon. Thus each hour the Earth rotates by $360/24 = 15$ degrees.

When at your location the time is 12 noon, 15° to the **east** the time is 1 p.m., for that is the meridian which faced the Sun an hour ago. On the other hand, 15° to the **west** the time is 11 a.m., for in an hour's time, **that** meridian will face the Sun and experience noon.

In the middle of the 19th century, each community across the US defined in this manner its own local time, by which the Sun, on the average, reached the farthest point from the horizon (for that day) at 12 o'clock. However, travelers crossing the US by train had to re-adjust their watches at every city, and long distance telegraph operators had to coordinate their times. This confusion led railroad companies to adopt **time zones**, broad strips (about 15° wide) which observed the same local time, differing by 1 hour from neighboring zones, and the system was adopted by the nation as a whole.

The continental US has 4 main time zones: eastern, central, mountain and western, plus several more for Alaska, the Aleut islands and Hawaii. Canadian provinces east of Maine observe Atlantic time; you may find those zones outlined in your telephone book, on the map giving area codes. Other countries of the world have their own time zones; only Saudi Arabia uses local times, because of religious considerations.

In addition, the clock is generally shifted one hour forward between April and October. This "**daylight saving time**" allows people to take advantage of earlier sunrises, without shifting their working hours. By rising earlier and retiring sooner, you make better use of the sunlight of the early morning, and you can enjoy sunlight one hour longer in late afternoon.

10.5 The Date Line and Universal Time (UT)

Suppose it is **noon** where you are and you **proceed west** — and suppose you could travel **instantly** to wherever you wanted.

Fifteen degrees to the west the time is 11 a.m., 30 degrees to the west, 10 a.m., 45 degrees–9 a.m. and so on. Keeping this up, 180 degrees away one should reach midnight, and still further west, it is the previous day. This way, by the time we have covered 360 degrees and have **come back to where we are**, the time should be noon again — **yesterday** noon. What happened?

We got into trouble because longitude determines only the hour of the day — **not the date**, which is determined separately. To avoid the sort of problem encountered above, the international date line has been established — most of it following the 180th meridian — where by common agreement, whenever we cross it the date advances one day (going west) or goes back one day (going east).

That line passes the Bering Strait between Alaska and Siberia, which thus have different dates, but for most of its course it runs in mid-ocean and does not inconvenience any local time keeping.

Astronomers, astronauts and people dealing with satellite data may need a time schedule which is the same everywhere, not tied to a locality or time zone. The **Greenwich mean time**, the astronomical time at Greenwich (averaged over the year) is generally used here. It is sometimes called **Universal Time (UT)**.

10.6 Right Ascension and Declination

The globe of the heavens resembles the globe of the Earth, and positions on it are marked in a similar way, by a network of **meridians** stretching from pole to pole and of **lines of latitude** perpendicular to them, circling the sky. To study some particular galaxy, an astronomer directs the telescope to its coordinates.

On **Earth**, the equator is divided into 360 degrees, with the zero meridian passing Greenwich and with the longitude angle ϕ measured east or west of Greenwich, depending on where the corresponding meridian meets the equator.

In the sky, the equator is also divided into 360 degrees, but the count begins at one of the two points where the equator cuts the **ecliptic** — the one which the Sun reaches around March 21. It is called the **vernal equinox** ("vernal" means related to spring in Latin) or sometimes the **first point in Aries**, because in ancient times, when first observed by the Greeks, it was in the zodiac constellation of Aries, the ram. It has since then moved, as is discussed in the later section on precession (http://www.phy6.org/stargaze/Sprecess.htm).

The celestial globe, however, uses terms and notations which differ somewhat from those of the globe of the Earth. Meridians are marked by the angle α (alpha, Greek A), called **right ascension**, not longitude. It is measured from the vernal equinox, but only eastward, and instead of going from 0 to 360 degrees, it is specified in hours and other divisions of time, each hour equal to 15 degrees.

Similarly, where on Earth latitude goes from 90° north to 90° south (or -90°), astronomers prefer the **co-latitude**, the angle from the polar axis,equal to 0° at the north pole, 90° on the equator, and 180° at the south pole. It is called **declination** and is denoted by the letter δ (delta, Greek small D). The two angles (α, δ), used in specifying (for instance) the position of a star are jointly called its **celestial coordinates**.

The next section tells how the stars, the Sun and accurate clocks allowed sailors to find their latitude and longitude.

10.7 Exploring Further

A site with star maps: http://csep10.phys.utk.edu/astr161/lect/time/maps.html

Image Sources

(1) Alex Zaliznyak, David Stern. *Coordinates*. CC-BY-SA 3.0.

Chapter 11

Navigation

I must go down to the sea again

To the lonely sea and sky

And all I ask is a tall ship

And a star to steer her by

***Sea Fever* by John Masefield**

How does a captain determine a ship's position in mid-ocean? In our space age, this is easily done, by using the GPS system of satellites–the Global Positioning System (http://www.phy6.org/stargaze/Satell5.htm#q150). That network of 24 satellites constantly broadcasts its positions, and small hand-held receivers exist which convert those signals into positions accurate within at least 15 meters or about 50 feet.

Before the space age, however, it was not as easy. One had to use **the Sun and the stars**.

11.1 Finding latitude with the Pole Star

Imagine yourself standing at night at point P on Earth and observing the pole star (or better, the position of the north celestial pole, near that star), at an elevation angle λ above the horizon.

The angle between the direction of the pole and the zenith is then $90° - \lambda$ degrees. If you continue the line from zenith downwards it reaches the center of the Earth, and the angle beween it and the Earth's axis is also $90° - \lambda$.

Therefore, as the accompanying figure shows, λ is also your **latitude**.

11.2 Finding latitude with the noontime Sun

If you are sailing a ship in mid-ocean, you can get the same information from the noontime Sun — probably more accurately, since at night you might not see the horizon very well.

Noon is when the Sun reaches the highest point in its journey across the sky. It then crosses the north-south direction–in the northern hemisphere, usually south of the observer. Because the axis of the Earth is inclined by an angle $e = 23.5°$ to a line perpendicular to the ecliptic, the height of that point above the horizon depends on the season. Suppose you are at point P. We examine 3 possibilities:

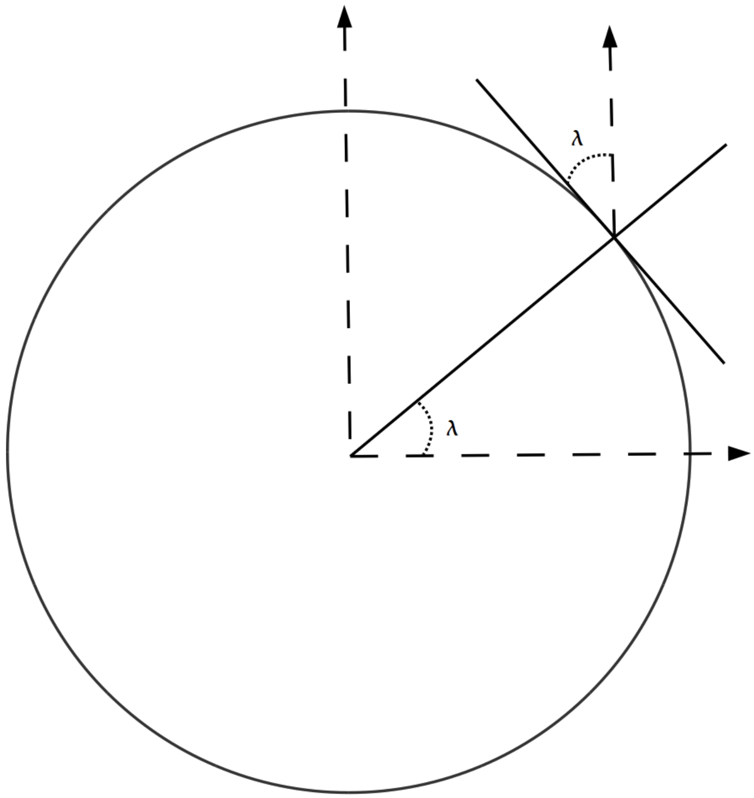

Figure 11.1: The angle of the pole star above the horizon equals the local latitude.

(1) Suppose the date is the winter solstice, around December 21, when the north pole is inclined away from the Sun. To find your latitude λ you measure the angle a between the direction of the noontime Sun and the zenith.

Look at the drawing and imagine you could rotate the equator and the north pole N until they reached the ecliptic and the pole of the ecliptic N'. Then all three angles marked e fold up together, showing that they are equal. You get

$$\mathrm{a} = \lambda + e$$

and your latitude is

$$\lambda = a - e = a - 23.5^\circ$$

(2) Half a year later, at the summer solstice (June 21), the north pole is inclined towards the Sun, not away from it, and now (if λ is larger than e)

$$\mathrm{a} = \lambda - e$$

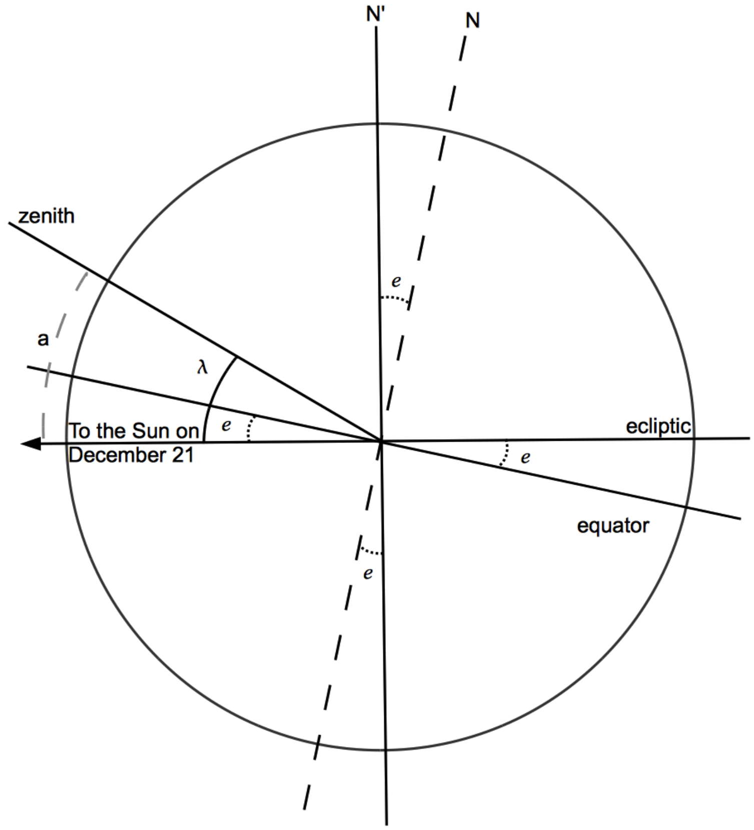

Figure 11.2: Position of the noon Sun at the winter solstice.

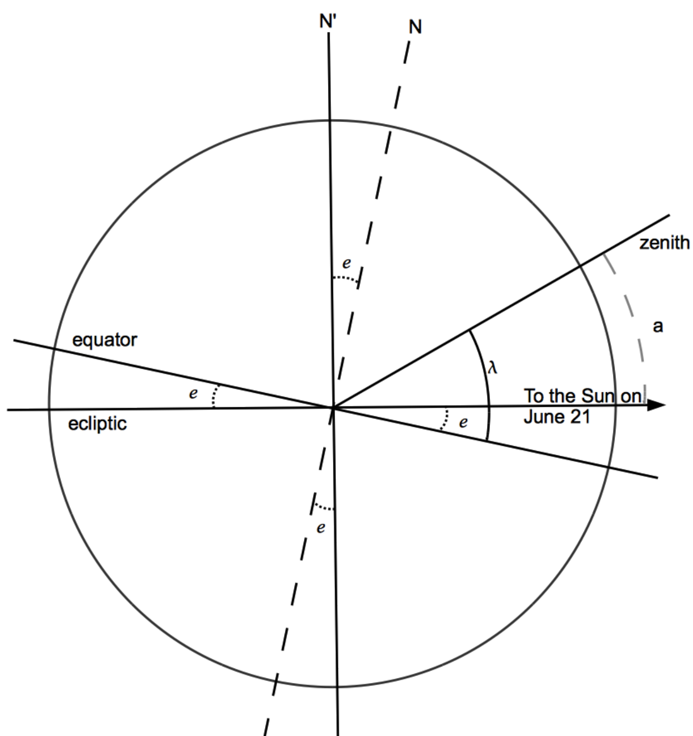

Figure 11.3: Position of the noon Sun at the summer solstice.

and your latitude is

$\lambda = a + e = a + 23.5°$

(3) Finally, suppose you are at equinox, around March 21 or September 21. The inclination of the Earth's axis is now out of the plane of the drawing — away from the paper, if this were a picture in a book.

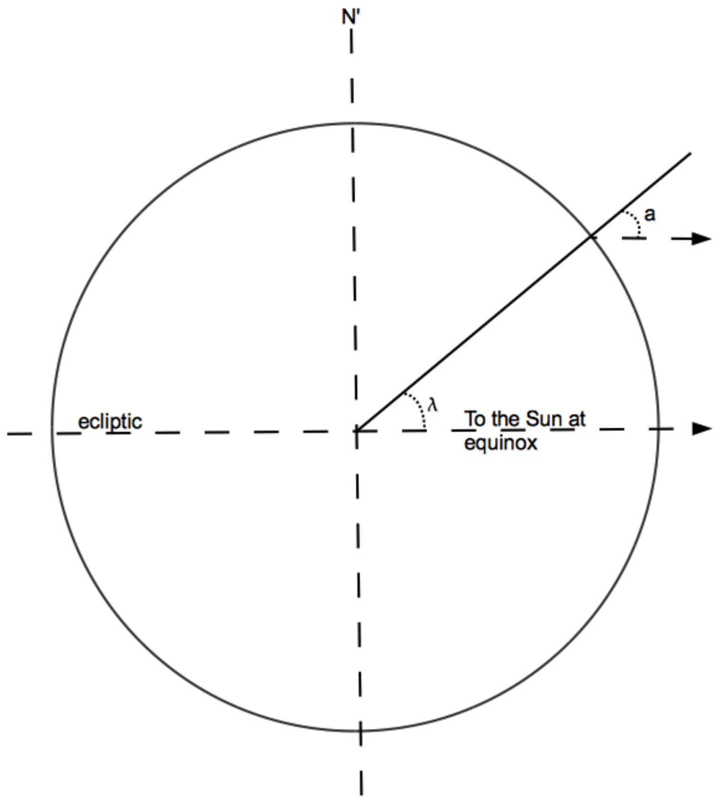

Figure 11.4: Position of the noon Sun at equinox.

The direction to the Sun is in the plane of the equator, and we get

$\lambda = a$

Thus at least at those dates, seafarers could tell what their latitude was by measuring the position of the noontime Sun.

For any other date, navigation tables exist that give the proper angle (smaller than 23.5 degrees) which must be added or subtracted. They also provide formulas for deriving the height of the noontime Sun from observations made at other times.

As with the pole star, rather than measuring the angle a from the zenith — which is not marked in the sky! — it is simpler to measure the angle ($90°–a$) from the horizon, which at sea is usually sharply defined. Such observations, known as "shooting the Sun," are done with an instrument known as the sextant. It has a sliding scale covering 1/6 of a circle (hence the name) and an attached pivoted mirror, providing a split view: by moving the scale, the sea-officer brings Sun and horizon simultaneously into view and then reads off the angle between them.

11.3 Longtitude

In the age of the great navigators — of Columbus, Magellan, Drake, Frobisher, Bering and others — finding your latitude was the easy part. Captains knew how to use the noontime Sun, and before the sextant was invented, a less precise instrument known as the cross-staff was widely used.

Longitude was a much harder nut to crack. In principle, all one needs is an accurate clock, set to Greenwich time. When the Sun "passes the meridian" at noon, we only need to check the clock: if Greenwich time is 3 p.m., we know that 3 hours ago it was noon at Greenwich and we are therefore at longitude $15^\circ \times 3 = 45$ degrees west.

However, accurate clocks require a fairly sophisticated technology. Pendulum clocks can keep time quite accurately on firm land, but the pitching and rolling of a ship makes them quite unsuitable for sea duty.

Non-pendulum clocks — e.g. wristwatches, before they became electronic — use a balance wheel, a small flywheel rotating back and forth through a small angle. A flat spiral spring is wrapped around its axis and it always brings the wheel back to its original position. The period of each back-and-forth oscillation is then only determined by the strength of the spring and the mass of the wheel, and it can replace the swing of the pendulum in controlling the motion of the clock's hands.

Gravity plays no role here, and motions of the ship also have very little effect; as discussed in a later section, a vaguely similar method was used in 1973 for "weighing" astronauts in the weightless environment of a space station. For navigation, however, such a clock must be **very** accurate, which is not easy to achieve: friction must be minimal, and so must changes in the dimensions of the balance wheel and properties of the spring due to changing temperature and other factors.

In the 17th and 18th century, when the navies of Britain, Spain, France and Holland all tried to dominate the seas, the "problem of longitude" assumed great strategic importance and occupied some of the best scientific minds. In 1714 Britain announced a prize of 20,000 pounds — a huge sum in those days — for a reliable solution, and John Harrison, a British clockmaker, spent decades trying to achieve it. His first two "**chronometers**," of 1735 and 1739, though accurate, were bulky and delicate pieces of machinery; they have been restored and are ticking away on public display, at the Royal Astronomical Observatory in Greenwich. Only his 4th instrument, tested in 1761, proved satisfactory, and it took some additional years before he received his prize.

11.4 Tales of Navigation #1: Robert Wood

Robert Wood was a professor at Johns Hopkins University during the first half of the 20th century, distinguished for his work on physical optics and also for his sense of humor and his love of mischievous tricks.

In September 1917, Wood and some colleagues embarked for Europe aboard the steamship Adriatic, to help US allies use science in fighting World War I. To hinder German submarines from intercepting the ship, it location was kept secret from everybody, including its passengers.

What follows are Wood's own notes, reproduced in "Doctor Wood" by William Seabrook (1940). The book is out of print, but remains worth reading (if you can find it) for its great store of stories, of which this one is a fair sample.

> "We sailed on night after night, the weather growing colder and colder, and the North Star climbing towards the zenith. One afternoon it occured to Colpitts [one of the traveling scientists] that it was the night of the autumnal equinox, on which both longitude and latitude can be calculated from the elevation of the North Star and the time of sunset [6 hours after noon]. I made a quadrant out of two sticks of wood and a protractor. By sighting one stick on the horizon and the other on the star, I determined its elevation, given which Colpitts, who had timed the sunset, worked out our position in a few minutes. This news spread rapidly, throwing the ship's officers into a frenzy, as all information regarding the course we were sailing was a dead secret. Next morning we discovered the ships' officers had set all of the clocks available to passengers three-quarters of an hour ahead, to confuse and baffle the scientists aboard."

The calculation which enabled Wood and Colpitts to determine the ship's position is described in the lesson plan provided for teachers and accompanying the present web page (http://www.phy6.org/stargaze/Lnavigat.htm).

11.5 Tales of Navigation #2: Nansen

Once radio arrived on the scene, early in the 20th century, the accuracy of chronometers became less critical, because broadcast time signals allowed shipboard timepieces to be reset periodically. But until then chronometers were essential to accurate navigation, as the following story illustrates.

In 1893 the Norwegian explorer Fridtjof Nansen set out towards the north pole (located in the ice-covered Arctic Ocean) in a specially strengthened ship, the "Fram." Having studied the currents of the Arctic Ocean, Nansen allowed "Fram" to be frozen into the polar ice, with which it slowly drifted across the water. Nearly two years, later, realizing that the course of "Fram" fell short of the pole, Nansen (who had prepared for this possibility) left the ship with his colleague Johansen and attempted to reach the pole by sleds over the ice. About 400 miles short of the pole they had to turn back: they wintered on a desolate island, in a hut they built of stones and walrus hides, and the following spring they headed for the islands of Svalbard (Spitzbergen).

They had been in the icy wilderness for more than a year, completely out of touch, but they always knew exactly where they were, because each man carried a spring-powered chronometer. Then disaster struck — in a moment of distraction, both forgot to rewind their chronometers and allowed them to run down. Suddenly, they were lost! Based on their last recorded positions, they made a guess and reset their timepieces, but the rest of their journey was clouded by uncertainty. Luckily, they did not have much further to go, and as chance had it, they encountered a British Arctic expedition which took them home. "Fram" broke free from the edge of the ice at about the same time; it is now on public display in Oslo.

Image Sources

(1) David Stern, Alex Zaliznyak. *Navigation 2*. CC-BY-SA 3.0.

(2) David Stern, Alex Zaliznyak. *Navigation 4*. CC-BY-SA 3.0.

(3) David Stern, Alex Zaliznyak. *Navigation 3*. CC-BY-SA 3.0.

(4) David Stern, Alex Zaliznyak. *Navigation 1*. CC-BY-SA 3.0.

Chapter 12

Coordinates

Coordinates are sets of numbers that describe **position** — position along a line, on a surface or in space. Latitude and longitude, or declination and right ascension, each is a system of coordinates on the **surface of a sphere** — on the globe of the Earth or the globe of the heavens.

12.1 Coordinates on a Flat Plane

A more widely used system are **cartesian coordinates**, based on a set of axes perpendicular to each other. They are named for **Rene Descartes** ("Day-cart"), a French scientist and philosopher who back in the 1600s devised a systematic way of labeling each point on a flat plane by a pair of numbers. You may well be already familiar with it.

The system is based on two straight lines ("axes"), perpendicular to each other, each of them marked with the distances from the point where they meet ("origin") — distances to the right of the origin and above it, the origin being taken as positive and on the other sides as negative.

Graphs use this system, as do some maps.

This works well on a flat sheet of paper, but the real world is 3-dimensional and sometimes it is necessary to label points in 3-dimensional space. The cartesian (x, y) labeling can be extended to 3 dimensions by adding a third coordinate z. If (x, y) is a point on the sheet, then the point (x, y, z) in space is reached by moving to (x, y) and then rising a distance z above the paper (points below it have negative z).

Very simple and clear, once a decision is made **on which side** of the sheet z is positive. By common agreement the positive branches of the (x, y, z) axes, in that order, follow the thumb and the first two fingers of the right hand when extended in a way that they make the largest angles with each other.

12.2 Polar Coordinates

Cartesian coordinates (x, y) are not the only way of labeling a point P on a flat plane by a pair of numbers. Other ways exist, and they can be more useful in special situations.

One system ("**polar coordinates**") uses the length r of the line OP from the origin to P (i. e. the distance of P distance to the origin) and the angle that line makes with the x-axis. Angles are often denoted by Greek letters, and here we follow conventions by marking it with θ. Note that while in the cartesian system x and y play very similar roles, here roles are divided: r gives distance and θ direction.

The two representations are closely related. From the definitions of the sine and cosine:

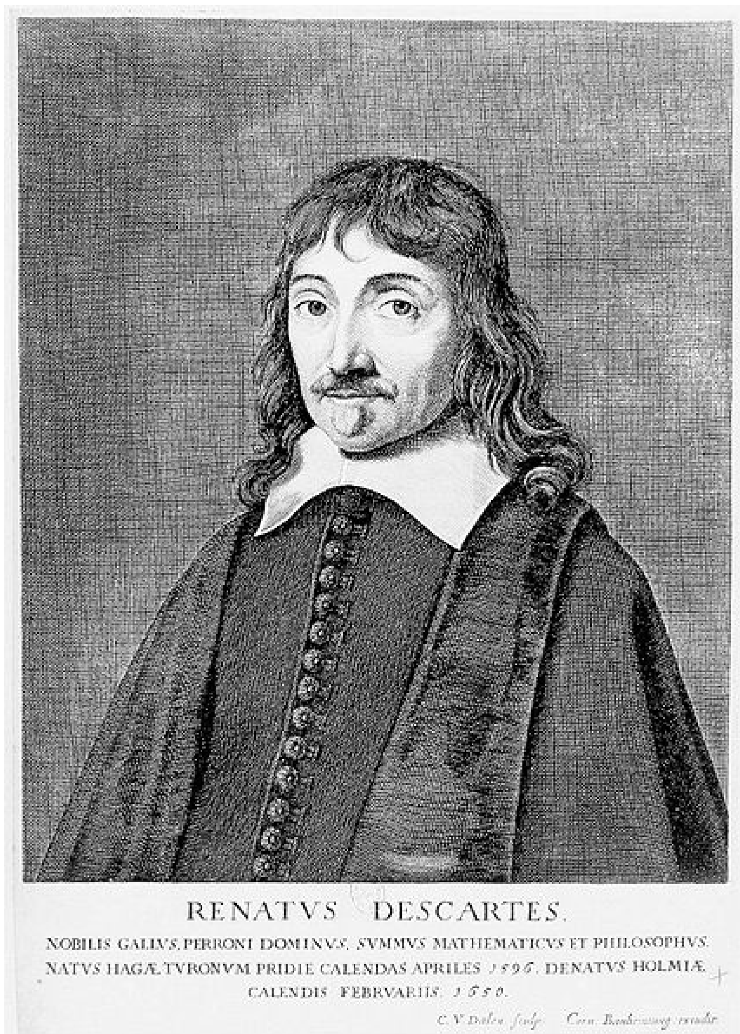

Figure 12.1: A portrait of Rene Descartes.

$\mathrm{x} = \mathrm{r} \times \cos\theta$
$y = r \times \sin\theta$

This allows (x, y) to be derived from polar coordinates. This relationship is illustrated below:

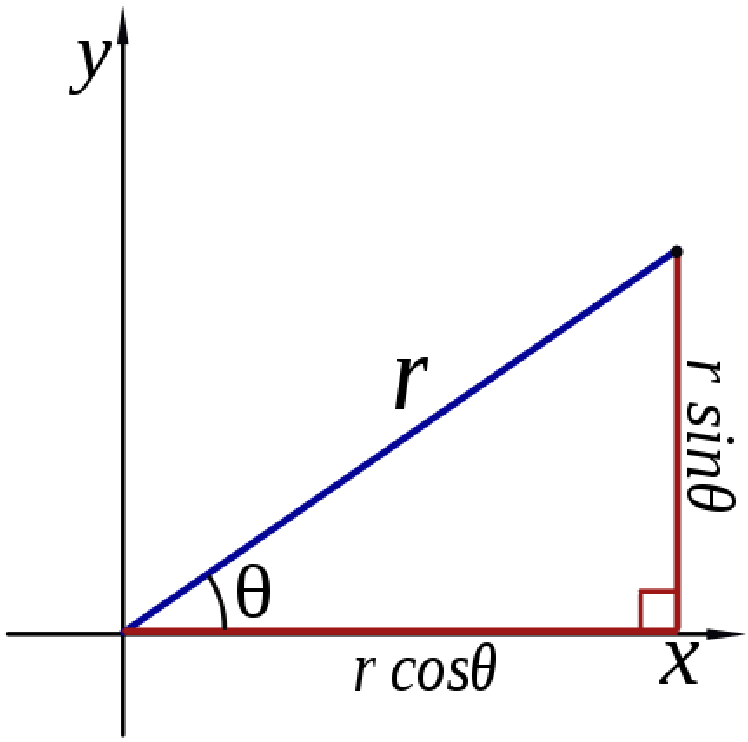

Figure 12.2: Converting from polar to cartesian coordinates.

To go in the reverse direction, we can use the Pythagorean theorem to find r:

$\mathrm{r}^2 = x^2 + y^2$

Once r is known, the rest is easy:

$\cos\theta = \frac{x}{r}$
$\sin\theta = \frac{y}{r}$

These relations fail only at the origin, where $x = y = r = 0$. At that point, θ is undefined and one can choose for it whatever one pleases.

In three dimensional space, the cartesian labeling (x, y, z) is nicely symmetric, but sometimes it is convenient to follow the style of polar coordinates and label distance and and direction separately. Distance is easy:

you take the line OP from the origin to the point and measure its length r. You can even show from the theorem of Pythagoras that in this case

$$r^2 = x^2 + y^2 + z^2$$

All the points with the same value of r form a **sphere** of radius r around the origin O. On a sphere we can label each point by latitude λ (lambda, small Greek L) and longitude ϕ (phi, small Greek F), so that the position of any point in space is defined by the 3 numbers (r, λ, ϕ).

12.3 Azimuth and Elevation

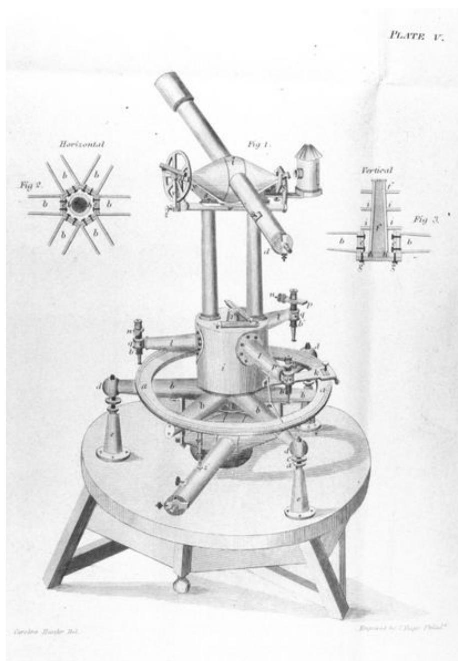

Figure 12.3: An old surveyor's telescope (theodolite).

The surveyor's telescope is designed to measure two such angles. The angle ϕ is measured counterclockwise in a horizontal plane, but surveyors (and soldiers) work with **azimuth**, a similar angle measured **clockwise** from **north**. Thus the directions (north, east, south, west) have azimuth (0°, 90°, 180°, 270°). A rotating table allows the telescope to be pointed in any azimuth.

The angle λ is called **elevation** and is the angle by which the telescope is lifted above the horizontal (if it looks down, λ is negative). The two angles together can in principle specify any direction: ϕ ranges from 0 to 360, and λ from –90 (straight down or "nadir") to +90 (straight up or "zenith").

Again, one needs to decide from what direction is the azimuth measured–that is, where is azimuth zero? The rotation of the heavens (and the fact most humanity lives north of the equator) suggests (for surveyor-type measurements) the northward direction, and this is indeed the usual zero point. The azimuth angle (viewed from the north) is measured counterclockwise.

Image Sources

(1) Cronholm144. *Rene Descartes.*. Creative Commons Attribution ShareAlike 3.0.

(2) unknown. *Rene Descartes.*. Public Domain.

(3) Caroline Hassler. *Theodolite*. Public Domain.

Chapter 13

The Cross Staff

Figure 13.1: The astronomer Ptolemy holding a cross staff.

The picture above is meant to represent the astronomer Claudius Ptolemy (an early champion of the geocentric model), who lived around 150 AD. It is an old picture, though not old enough for the artist to have actually known what Ptolemy looked like. **But what is that gentleman holding?**

It isn't a religious symbol — the proportions are not right, and the marks on the stick do not seem appropriate. It is actually a **cross staff** (or "Jacob's staff"), a tool widely used by astronomers and navigators before the invention of the telescope, and for a while afterwards. It consists of a main staff with a perpendicular crosspiece, attached at its middle to the staff and able to slide up and down along it.

The device was likely invented by **Rabbi Levi ben Gershon** (1288-1344), a Jewish scholar who lived in Provence, in southern France, also referred to as "Gersonides." Claudius Ptolemy lived more than 1000 years earlier, so the drawing on top indeed takes considerable artistic license.

Astronomers used the cross-staff for measuring the angle between the directions of two stars. Other, older instruments for this purpose existed, used by scholars such as Hipparchos and Ptolemy, but none was as portable, which made the cross-staff eminently suitable for navigation at sea. **Ships' officers** used it to measure the elevation angle of the noontime Sun above the horizon, which allowed them to estimate their latitude (see section on navigation). The problem of getting dazzled by the Sun later led to the invention of the *'backstaff*, where the sunlight fell onto a target, not into the eye. Columbus may well have used one. This was greatly improved around 1594 by Captain John Davis, so perhaps the "Mayflower" used the upgraded design.

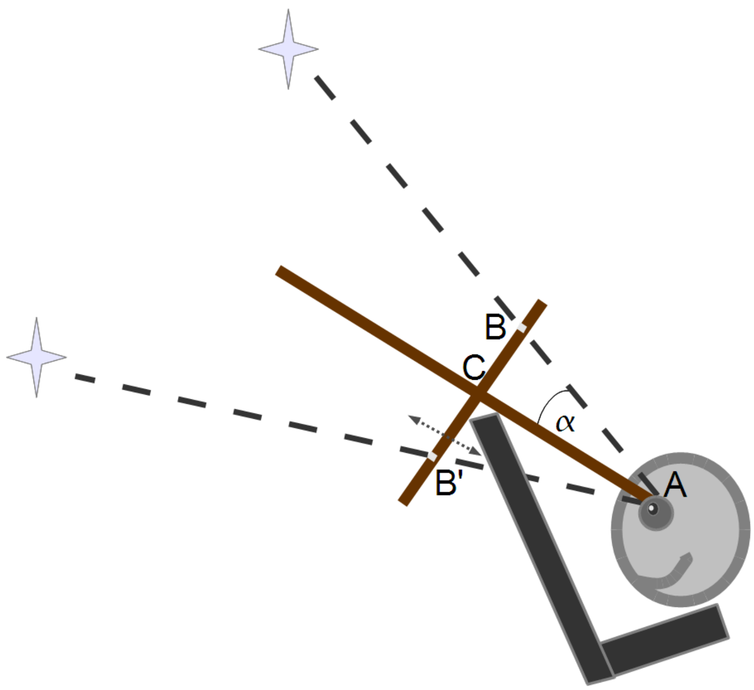

Figure 13.2: Illustration of a cross-staff.

To measure the angle between two stars, an astronomer would place the staff just below one eye (see **Figure** 13.2) and slide the cross-piece up and down. The cross-piece would have a pair of open sights sticking out perpendicular to the drawing at symmetric locations such as B and B' (often several pairs of sights, some spaced further apart than others). The astronomer would slide the cross-piece up and down, until sight B covered one of the stars and sight B' the other. For use at night, slits make convenient sights.

After that was achieved, the instrument would be lowered and the distance AC would be measured. Then if α was the angle between the staff and the direction of one star, from the definition of the tangent:

$$\tan(\alpha) = \frac{BC}{AC}$$

The distance BC between the sight and the stick was already known to the astronomer — so, using a table of tangents, the angle denoted by α could be calculated. Since the instrument was symmetric, the angle between the directions of the stars was 2α.

Image Sources

(1) Alex Zaliznyak, David Stern. *Cross-Staff*. CC-BY-SA 3.0.

(2) Andre Thevet (1502-1590). *Ptolemy and His Cross Staff.*. Public Domain.

Chapter 14

The Calendar

So familiar has the calendar become that people tend to forget that it, too, had to be invented. Early farmers needed to know when to plow and sow ahead of rainy seasons, and to time other seasonal activities. Early priests in Babylonia, Egypt, China and other countries, even among the Maya in America, examined therefore the motions of the Sun, Moons and planets across the sky, and came up with a variety of calendars, some still in use.

14.1 The Day

The basic unit is obviously the day: 24 hours, 1440 minutes, 86400 seconds, each second slightly longer than the average heartbeat. The day is defined by the motion of the Sun across the sky, and a convenient benchmark is **noon**, the time when the Sun is at its highest (i. e. most distant from the horizon) and is also exactly south or north of the observer.

"One day" can therefore be conveniently defined as the time from one noon to the next. A **sundial** can track the Sun's motion across the sky by the shadow of a rod or fin ("gnomon") pointing to the celestial pole (See the chapter "Making a Sundial" in the *From Stargazers to Starships FlexBook* on www.ck12.org for construction of a folded-paper sundial), allowing the day to be divided into hours and smaller units. Noon is the time when the shadow points exactly south (or north) and is at its shortest.

What then is the period of the Earth's rotation around its axis? A day, you might guess? Not quite.

Suppose we observe the position of a **star** in the sky — for instance Sirius, the brightest of the lot. One full rotation of the Earth is the time it takes for the star to return to its original position (of course, we are the ones that move, not the star). That is almost how the day is defined, but with one big difference: for the day, the point of reference is not a star fixed in the firmament, but the Sun, whose position in the sky slowly changes. During the year the Sun traces a full circle around the sky, so that if we keep a separate count of "Sirius days" and "Sun days," at the end of the year the numbers will differ by 1. We will get 366. 2422 "star days" but only 365. 2422 Sun days.

It is the "star day" (sidereal day) which gives the rotation period of the Earth, and it is about 4 minutes shy of 24 hours. A clockwork designed to make a telescope follow the stars makes one full rotation per sidereal day.

The clocks we know and use, though, are based on the solar day — more precisely, on the **average** solar day, because the time from noon to noon can vary as the Earth moves in its orbit around the Sun. By Kepler's laws (discussed in a later section:http://www.phy6.org/stargaze/Skeplaws.htm) that orbit is slightly elliptical. The distance from the Sun therefore varies slightly, and by Kepler's second law, the motion speeds up when nearer to the Sun and slows down when further away. Such variations can make "sundial time" fast or slow, by up to about 15 minutes.

Very precise atomic clocks nowadays tell us that the day is gradually getting longer. The culprits are the **tides**, twin waves raised in the Earth's ocean by (mainly) the Moon's gravitational pull. As the waves travel around the Earth, they break against shorelines and shallow seas, and thus give up their energy: theory suggests that this energy comes out of the (kinetic) energy of the Earth's rotational motion.

14.2 The Year

The year is the time needed by the Earth for one full orbit around the Sun. At the end of that time, the Earth is back to the same point in its orbit, and the Sun is therefore back to the same apparent position in the sky.

It takes the Earth 365. 2422 days to complete its circuit (average solar days), and any calendar whose year differs from this number will gradually wander through the seasons. The ancient Roman calendar had 355 days but added a month every 2 or 4 years: it wasn't good enough, and by the time Julius Caesar became ruler of Rome, it had slipped by three months.

In 46 BC Caesar introduced a new calendar, named after him: the **Julian calendar**. It is similar to the one used today: the same 12 months, and an added day at the end of February every 4th year ("leap year"), on years whose number is divisible by 4. Two years afterwards the 5th month of the Roman year was renamed July, in honor of Julius. The name of his successor, Augustus Caesar, was later attached to the month following July.

The Julian calendar thus assumes a year of 365. 25 days, leaving unaccounted a difference of 0. 0078 days or about 1/128 of a day. Thus the calendar still slips, but at a very slow rate, about one day in 128 years. By 1582 that slippage was approaching two weeks and Pope Gregory the 13th therefore decreed a modified calendar, named after him: the **Gregorian calendar**. Henceforth years ending in two zeros, such as 1700, 1800, 1900–would not be leap years, except when the number of centuries was divisible by 4, such as 2000. This took away 3 "leap days" every 400 years, i. e. one day per 133 1/3 years — close enough to the required correction of one day per 128 years.

But it was not enough to modify the calendar: a one-time jump of dates was also needed, to get rid of the accumulated difference. In Italy this was done soon after the pope's edict, and "Tibaldo and the Hole in the Calendar" by Abner Shimony spins the story of a boy whose birthday was on a day skipped by that jump. Another birthday affected was that of George Washington, born February 11, 1732: when the British empire in September 1752 implemented the Gregorian calendar, the 11th of February "old style" became the 22nd of February "new style," and nowadays that is when Washington's birthday is usually celebrated.

In Russia the change came only after the revolution, which is why the Soviet government used to celebrate the anniversary of the "October Revolution" on November 7th. The Russian orthodox church continues to use the Julian calendar and celebrates Christmas and Easter about 2 weeks later than most of the Christian world.

14.3 The Moon

The Moon's orbital period, measured by the stars ("sidereal period") is 27. 321662 days. However, the monthly cycle of the Moon — thin crescent to half-moon, to full and back to crescent — takes 29. 530589 days, because it depends on the position of the Sun in the sky, and that position changes appreciably in the course of each orbit. The different shapes of the Moon represent different angles of illumination, and the appearance of the Moon in the night sky gives a fair idea of where the Sun would be (e. g. the Moon observed in the east before sunrise appears illuminated from below). The duration of the Moon's cycle ("synodic period") gave rise to the division of time known as **month**.

Many ancient calendars were based on the month. The most successful of these is the "Metonic" calendar, named after the Greek Meton, who noted that adding 7 months in the course of 19 years kept the calendar almost exactly in step with the seasons. That would make the length of the average year (12 + 7/19) months, and with a calculator you can easily find its value as

$$(12+7/19)\times 29.530589=365.2467 \text{ days,}$$

pretty close to the full value 365. 2422. The Metonic calendar is thus more accurate than the Julian one, though less so than the Gregorian. It is still used by Jews, on whose calendar each month begins at or near the new moon, when the Moon's position in the sky is nearest to the Sun's. The traditional Chinese calendar also uses of a formula like Meton's, which was probably invented by the ancient Babylonians. For more about the ancient Babylonian calendar see http://en.wikipedia.org/wiki/Babylonian_calendar.

14.4 The Islamic Calendar

Muslims use an uncorrected lunar calendar, and as a result their holidays slip through the seasons at a rate of about 11 days per year. The reason is not ignorance of astronomy, but a deliberate effort to follow a different schedule from that of any other faith.

This creates an interesting situation during the month of Ramadan, when faithful Muslims are expected to fast and abstain from drinking from sunrise to sunset. When Ramadan falls in mid-winter, this imposes no great hardship, since days are short and cool. Fifteen years later, however, Ramadan falls in mid-summer, when days are long and the heat makes people quite thirsty. That is when many in Arab cities wait impatiently for the boom of the cannon which traditionally announces every evening the end of the fast.

14.5 The Persian Calendar

> Ah, but my Computations, People say
>
> Reduced the Year to better reckoning – Nay,
>
> 'Twas only striking from the Calendar
>
> Unborn To-morrow and dead Yesterday
>
> ***Rubaiyat, verse #57*, by Omar Khayyam**

A calendar which tracks the solar year even better than the Gregorian one is the Persian (Iranian) calendar, the first version of which was devised by Omar Khayyam (1044-1123), author of the famous "Rubaiyat" poems, masterfully translated in 1839 into English by Edward Fitzgerald. It is also called the **Jalali** calendar, after the king Malik Shah Jalaludin who in 1074 assigned Omar and 7 other scholars to devise a new calendar.

Though the count of Persian years starts, like the Islamic one, from the flight of Mohammed to Medina in 622, establishing there the first strong base of Islam, the new year starts at the spring equinox, March 21, with the holiday of Nowruz.

The Persian year itself has 12 months — the first 6 have 31 days, the next 5 have 30 days, and the last has 28 or 29, depending on whether the year is or isn't a leap year. Each month corresponds to a sign of the zodiac. The number of days in each month (if not the order of months) is therefore the same as in the Western civil calendar. The difference is in the rule for determining leap year, which is more complex. Even the original Jalali calendar was more accurate than the Gregorian one; the current version assigns 683 leap years in a cycle of 2820 years and would take two million years before it shows a one-day inaccuracy!

An interesting related calendar is used by the Coptic Christian church in **Ethiopia**, with 12 months of 30 days each, plus a 13th short month of 5 days. A tourist brochure once lured visitors with a promise "Come to Ethiopia and enjoy 13 months of sunshine a year."

14.6 The Maya Calendar

The Maya Indians in Central America, living on the Yucatan peninsula in Mexico, Belize and Guatemala (where Maya languages are still spoken), created an extensive civilization which peaked around the years 1200-1450. They developed an early system of symbolic writing ("glyphs") and mathematics, using a system like ours (including the zero!) but based on the number 20 instead of 10. They did not, however, use fractions.

Their astronomy was well developed, and they noted the "zenial days" when the Sun was directly overhead ("at zenith") and a vertical stick cast no shadow. Their year had 365 days, but in the absence of leap years it slowly shifted with respect to the solstices. That year was divided into 18 named "months" of 20 days each (numbered from 0 to 19), plus the "short month" of Wayeb, whose days were considered unlucky.

Yucatan does not experience summer and winter the way middle latitudes do (e.g. Europe or most of the US), and therefore the Maya calendar was not strongly tied to the seasons the way ours is. The planet Venus received major attention, and its cycles were accurately measured by Maya astronomers. In addition the Maya also observed a "ritual year" of 260 days, consisting of 20 named "long weeks" of 13 numbered days each.

14.7 Exploring Further

Here's a series of pages devoted to different calendars: `http://webexhibits.org/calendars/calendar-mayan.html`

Another inventory of calendars: `http://www.projectpluto.com/calendar.htm`

About Julius Caesar and leap days `http://antwrp.gsfc.nasa.gov/apod/ap960229.html`

"**Tibaldo and the Hole in the Calendar**" by Abner Shimony, 165 pp, Copernicus 1998. The book tells the story of a boy in 16th-century Italy whose birthday celebration was set for one of the "lost" days, skipped over by the one-time jump in the calendar which Pope Gregory the 13th ordered. Reviewed by Stephen Battersby in Nature, p. 460, 3 April 1998, and by David Mermin in Physics Today, p. 63, June 1998.

Wikipedia has an extensive page on the Maya Calendar: `http://en.wikipedia.org/wiki/Maya_calendar`

To learn more about the **Maya and Venus** in informal writing, see the chapter "Bringing Culture to the Physicists," p. 313 in "Surely You're Joking, Mr. Feynman!" by the great physicist Richard Feynman.

Chapter 15

Precession

The priests of ancient Babylonia and Egypt were also pioneer astronomers. They studied the heavens, mapped their constellations, identified the path of the Sun and estimated the periods of the Moon and Sun as they moved across the sky.

But it was a Greek astronomer, **Hipparchus of Nicea**, who made the first major new discovery in astronomy. Comparing observations more than a century apart, Hipparchus proposed that the axis around which the heavens seemed to rotate **shifted** gradually, though very slowly.

Viewed from Earth, the Sun moves around the ecliptic, one full circuit each year. Twice a year, at **equinox**, day and night are equal and the Sun rises exactly in the east and sets exactly in the west. Ancient astronomers had no good clocks and could not tell when the day and night had the same length, but they could identify the equinox by the Sun rising exactly in the east and setting exactly in the west. At those times the Sun's position is at one of the intersections between the ecliptic and the celestial equator.

Around the year 130 BC, Hipparchus compared ancient observations to his own and concluded that in the preceding 169 years those intersections had **moved by 2 degrees**. How could Hipparchus know the position of the Sun among the stars so exactly, when stars are not visible in the daytime? By using not the Sun but the **shadow cast by the Earth on the moon**, during an eclipse of the Moon! During an eclipse, Sun, Earth and Moon form a straight line, and therefore the center of the Earth's shadow is at the point on the celestial sphere which is **exactly opposite** that of the Sun.

15.1 The Dawning of the Age of Aquarius

Hipparchus concluded that the intersection marking the equinox slowly crept forward along the ecliptic, and called that motion "**the precession of the equinoxes**." The rate is about one full circle in 26 000 years. In ancient times the intersection marking the spring equinox was in the constellation of **Aries**, the ram, and for that reason the intersection (wherever it might be) is still sometimes called "the first point in Aries."

Around the year 1 it moved into the constellation of **Pisces** (pronounced "pie-sees" in the US) and currently it is again in transition, to the constellation of **Aquarius**, the water carrier. If you ever heard the song "The dawning of the age of Aquarius" from the musical "Hair," that is what it is all about. To astronomers precession is mainly another factor to be taken into account when aiming a telescope or drawing a star chart; but to believers in astrology, the "dawning of the age of Aquarius" is a great portent and may mark the beginning of a completely new and different era.

15.2 The Precession of the Earth's Axis

What does this motion tell us about the Earth's motion in space? If you ever had a spinning top, you know that its axis tends to stay lined up in the same direction — usually, vertically (left figure), though in space any direction qualifies.

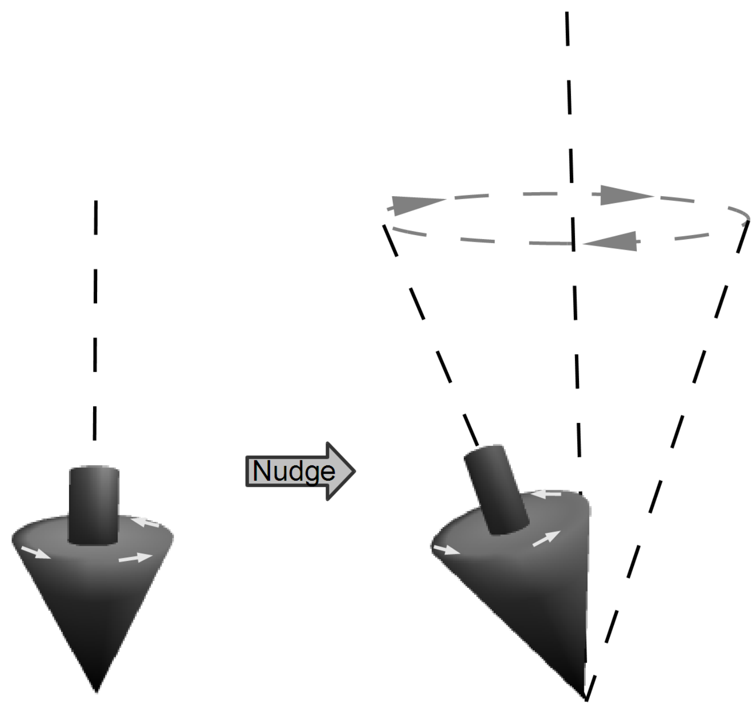

Figure 15.1: Precession of a spinning top: the spin axis traces the surface of a cone.

Give it a nudge, however, and the axis will start to gyrate wildly around the vertical, its motion tracing a cone (right figure). The spinning Earth moves like that, too, though the time scale is much slower — each spin lasts a day, and each gyration around the cone takes 26 000 years. The axis of the cone is perpendicular to the plane of the ecliptic.

The cause of the precession is the equatorial bulge of the Earth, caused by the centrifugal force of the Earth's rotation (the centrifugal force is discussed in a later section). That rotation changes the Earth from a perfect sphere to a slightly flattened one, thicker across the equator. The attraction of the Moon and Sun on the bulge is then the "nudge" which makes the Earth precess.

Through each 26 000-year cycle, the direction in the sky to which the axis points goes around a big circle, the radius of which covers an angle of about 23.50. The pole star to which the axis points now (within about one degree) used to be distant from the pole, and will be so again in a few thousand years (for your information, the closest approach is in 2017). Indeed, the "pole star" used by ancient Greek sailors was a different one, not nearly as close to the pole.

Because of the discovery mady by Hipparchus, the word "precession" itself no longer means "shift forward" but is now applied to any motion of a spin axis around a cone–for instance, the precession of a gyroscope in an airplane's instrument, or the precession of a spinning satellite in space.

Precession of a spinning scientific payload (also known as its "coning" — from "cone" — or its "nutation") is an unwelcome feature, because it complicates the tracking of its instruments. To eliminate it, such

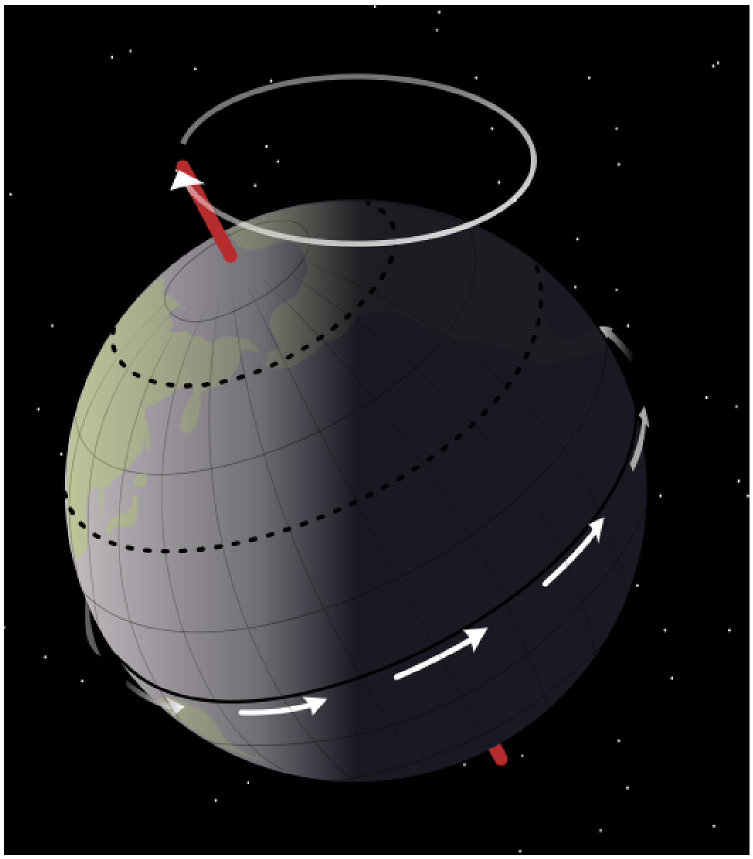

Figure 15.2: Precession of the Earth's axis.

satellites use "nutation dampers," small tubes partially filled with mercury. If the satellite spins as it was designed to do, the mercury merely flows to the part of the tube most distant from the spin axis, and stays there. However, if the axis of rotation precesses, the mercury sloshes back and forth in the tube. Its friction then consumes energy, and since the source of the sloshing is the precession of the spin axis, that precession (very gradually) loses energy and dies down.

> **Note**: In the section on the calendar, we saw that the Earth's rotation is slowed down very gradually by the tides, raised by the gravity of the Moon. That process is a bit similar to the action of nutation dampers: the energy of the tides is "lost"–that is, converted to heat–when the waves caused by tides break up on the seashore, and that loss is ultimately taken away from the rotational motion (**not** the precession) of the Earth.

15.3 Ice Ages

> Some say the world will end in fire,
>
> Some say in ice.
>
> From what I've tasted of desire
>
> I hold with those who favor fire.
>
> But if it had to perish twice,
>
> I think I know enough of hate
>
> To say that for destruction ice
>
> Is also great
>
> And would suffice
>
> **Robert Frost**

Some 2000 years after Hipparchus, in the year 1840. Louis Agassiz, a Swiss scientist, published a book on glaciers, a familiar feature of his homeland — huge rivers of ice created by accumulated snowfall, filling valleys and slowly creeping downwards to their end points, lakes of meltwater (or, in some other countries, the sea).

Glaciers leave an imprint on the landscape: they scratch and grind down rocks, and carry loads of gravel, at times even big boulders, from the mountains to the plains, leaving them far from their origins, wherever the ice finally melts. Agassiz, who later became a distinguished professor at Harvard, noted that such imprints existed all over northern Europe, and suggested that the lands now inhabited by Germans, Poles, Russians and others used to be covered by enormous glaciers.

America, too, had its glaciers; Cape Cod, for instance, is a left-over pile of glacial gravel. Later geological studies found evidence that such glaciers advanced and retreated several times in the last million years. The last retreat, a rather abrupt one, occured about 12,000 years ago.

15.4 The Milankovich Theory

The big questions are, of course, what caused those glaciers to spread, and will it happen again? Actually, no one is yet completely sure. But an intriguing idea, due to work in the 1930s by the Serbian astronomer Milutin Milankovich, may link them to the precession which Hipparchus discovered.

Figure 15.3: Aletsch glacier in Switzerland.

As already noted, the Earth's orbit is not perfectly round, but is slightly elongated. The Earth therefore comes closest to the Sun in the first week of January (the exact day varies a little). It means that just when the northern hemisphere experiences winter and receives the **least** amount of sunlight, the Earth as a whole receives the **most** (the swing is about 3%, peak to peak). This makes northern winters milder, and northern summers are milder too, since they occur when the Earth is most distant from the Sun.

The opposite is true south of the equator: the beginning of January occurs there in summer, and therefore one expects southern summers to be hotter, and southern winters colder, than those north of the equator. This effect is however greatly weakened, because by far most of the the southern hemisphere is covered by ocean, and the water tempers and moderates the climate.

Right now, northern winter occurs in the part of the Earth's orbit where the north end of the axis points away from the Sun. However, since the axis moves around a cone, 13,000 years from now, in this part of the orbit, it will point towards the Sun, putting it in mid-summer just when the Earth is closest to the Sun.

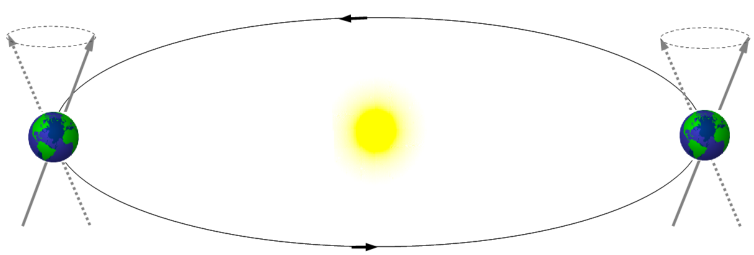

Figure 15.4: The Earth's current spin axis (solid) and its axis in 13,000 years (dashed).

At that time one expects northern climate to be more extreme, and the oceans then have a much smaller effect, since the proportion of land in the northern hemisphere is much larger. Milankovich argued that because winters were colder, more snow fell, feeding the giant glaciers. Furthermore, he said, since snow was white, it reflected sunlight, and with more severe winters, the snow-covered land warmed up less effectively once winter had ended. Climate is maintained by a delicate balance between opposing factors, and Milankovich argued that this effect alone was enough to upset that balance and cause ice ages.

Milankovich was aware that this was just one of several factors, since it turns out that **ice ages do not recur every 26,000 year**, nor do they seem common in other geological epochs. The eccentricity of the Earth's orbit, which determines the closest approach to the Sun, also changes periodically, as does the inclination of the Earth's axis to the ecliptic. But overall the notion that ice ages may be linked to the motion of the Earth through space may be currently our best guess concerning the causes of ice ages.

> **Postscript, 28 July 1999**. The magnitude of the "Milankovich effect" depends on the difference between largest and smallest distances from the Sun. That, in its turn, depends on the eccentricity of the Earth's orbit, which varies with a 100,000-year cycle, on which a 413,000-year cycle is superposed. J. Rial (Univ. of North Carolina) found signatures of those cycles in the oxygen isotope content of deep-sea sediments, in full agreement with the Milankovich theory. His work is in ***Science***, vol. 285, p. 564, 23 July 1999; a non-technical explanation "Why the Ice Ages Don't Keep Time " is on pages 503-504 of the same issue.
>
> **Further note**: The sea-bottom results have now been compared to hydrogen isotope ratios in deep boreholes in the ice sheets of Antarctica, which took nearly a million years to accumulate (***Science***, 11 June 2004, p. 1609). Deep-sea sediments show that in the last million years, but not before, the variation is dominated by a periodicity around 100,000 years. Its origin, the article states, "is one of the unanswered, yet fundamental questions." Ice cores could help explain it.

15.5 Exploring Further

Because of the precession of the equinoxes, the position among the stars of the celestial pole — the pivot around which the celestial sphere seems to rotate — traces a circle every 26,000 years of so. The celestial pole is now quite close to the pole star Polaris, but it will not be so in the future, and wasn't in the past. The ancient Egyptians regarded as pole star the star Thuban (http://www.astro.uiuc.edu/~kaler/sow/thuban.html) or "Alpha Draconis," the brightest star (=alpha) in the constellation Draco, the serpent.

A review article, primarily for scientists: **Trends, Rhythms and Aberrations in Global Climate 65 Ma to the Present** (Ma is million years), by James Zachos, Mark Pagani, Lisa Sloan, Ellen Thomas and Katharina Billups, ***Science*** vol 292, p. 686, 27 April 2001. Goes beyond variations due to the precession of the equinoxes and also includes variations of orbit eccentricity, inclination between spin axis and the ecliptic and in the precession cycle itself.

The full scoop on the Milankovich theory (including other periodicities) can be found here: http://deschutes.gso.uri.edu/~rutherfo/milankovitch.html

Wikipedia's biography of Milankovich can be found at http://en.wikipedia.org/wiki/Milutin_Milankovi%C4%87

For serious scientific pursuits: A recent article in ***Nature*** applies the theory of Milankovich to Mars and concludes that its effects there were probably much more severe, in part because the presence of our Moon regulates the tilt angle of the Earth's rotation axis. See Recent ice ages on Mars by James W. Head et al., Nature, vol 426, p, 797-802, 18/25 December 2003.

Image Sources

(1) NASA, Mysid. *Precession of the Earth's axis.*. Public Domain.

(2) Alex Zaliznyak. *Precession of a Top.*. cc-by-sa-3.0.

(3) Dirk Beyer. *Aletsch Glacier*. cc-by-sa-2.5.

(4) Alex Zaliznyak. *Precession of the Earth's axis 2.*. cc-by-sa-3.0.

Chapter 16

The Round Earth and Columbus

Today it is well known that the Earth is a sphere, or very close to one (its equator bulges out a bit because of the Earth's rotation). When Christopher Columbus proposed to reach India by sailing west from Spain, he too knew that the Earth was round. India was the source of precious spices and other rare goods, but reaching it by sailing east was difficult, because Africa blocked the way. On a round globe, however, it should also be possible to reach India by sailing west, and this Columbus proposed to do (he wasn't the first one to suggest this—see below).

Sometimes the claim is made that those who opposed Columbus thought the Earth was flat, but that wasn't the case at all. Even in ancient times sailors knew that the Earth was round and scientists not only suspected it was a sphere, but even estimated its size.

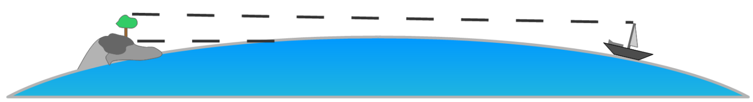

Figure 16.1: A ship not visible from the shore might be visible from a tree

If you stand on the seashore and watch a ship sailing away, it will gradually disappear from view. But the reason cannot be the distance: if a hill or tower are nearby, and you climb to the top after the ship has **completely** disappeared, it becomes visible again. Furthermore, if on the shore you watch carefully the way the ship disappears from view, you will notice that the hull vanishes first, while the masts and sails (or the bridge and smokestack) disappear last. It is as if the ship was dropping behind a hill, which in a way is **exactly** the case, the "hill" being the curve of the Earth's surface.

To find out how the distance to the horizon is calculated, please see the chapter "Distance to the Horizon" in the *From Stargazers to Starships FlexBook* on www.ck12.org.

16.1 Eratosthenes, Posidonius and El Mamun

The Greek philosopher Aristotle (384-322 BC) argued in his writings that the Earth was spherical, because of the circular shadow it cast on the Moon, during a lunar eclipse. Another reason was that some stars visible from Egypt are not seen further north The full quotation can be found at http://www.phy6.org/stargaze/Saristot.htm

The Alexandria philosopher http://en.wikipedia.org/wiki/Eratosthenes Eratosthenes went one step further and actually estimated how large the Earth was. He was told that on midsummer day (June 21) in the town of Syene in southern Egypt (today Aswan, near a huge dam on the river Nile) the noontime Sun was reflected in a deep well, meaning that it was right overhead, at zenith. Eratosthenes himself lived in Alexandria, near the river's mouth, north of Syene, about 5000 stadia north of Syene (the stadium, the size of a sports arena, was a unit of distance used by the Greeks). In Alexandria the Sun on the corresponding date did not quite reach zenith, and vertical objects still threw a short shadow. Eratosthenes established that the direction of the noon Sun differed from the zenith by an angle that was 1/50 of the circle, that is, 7.2 degrees, and from that he estimated the circumference of the Earth to be 250,000 stadia.

Tidbit Eratosthenes also headed the royal library in Alexandria, the greatest and most famous library in classical antiquity. Officially it was called "temple of the muses" or "museion," from which our modern "museum" is derived.

Other estimates of the size of the Earth followed. Some writers reported that the Greek Posidonius used the greatest height of the bright star Canopus above the horizon, as seen from Egypt and from the island of Rhodes further north (near the southwestern tip of Turkey). He obtained a similar value, a bit smaller. The Arab Khalif El Ma'mun, who ruled in Baghdad from 813 to 833, sent out two teams of surveyors to measure a north-south baseline and from it also obtained the radius of the Earth. Compared to the value known today, those estimates were pretty close to the mark.

The idea of sailing **westward** to India dates back to the early Romans. According to Dr. Irene Fischer, who studied this subject, the Roman writer Strabo, not long after Erathosthenes and Posidonius, reported their results and noted:

> *"if of the more recent measurements of the Earth, the one which makes the Earth smallest in circumference be introduced—I mean that of Posidonius who estimates its circumference at about 180,000 stadia, then. . . "*

and he continues:

> *"Posidonius suspects that the length of the inhabited world, about 70,000 stadia, is half the entire circle on which it had been taken, so that if you sail from the west in a straight course, you will reach India within 70,000 stadia. "*

Notice that Strabo—for unclear reasons—reduced the 250,000 Stadia of Eratosthenes to 180,000, and then stated that half of that distance came to just 70,000 stadia. Handling his numbers in that loose fashion, he could argue that India was not far to the west.

16.2 Columbus Again

All these results were known to the panel of experts which King Ferdinand appointed to examine the proposal made by Columbus. They turned Columbus down, because using the original value by Eratosthenes, they calculated how far India was to the west of Spain, and concluded that the distance was far too great.

Columbus had an estimate of his own. Some historians have proposed that he used an argument like Strabo's, but Dr. Fischer found his claim to be based on incorrect units of distance. Columbus used an erroneous estimate by Ptolemy (whom we meet again), who based it on a later definition of the stadium, and in estimating the size of the settled world he confused the Arab mile, used by El Ma'mun, with the Roman mile on which our own mile is based. All the same, his final estimate of the distance to India was close to Strabo's.

In the end Queen Isabella overruled the experts, and the rest is history. We may never know whether Columbus knowingly fudged his values to justify an expedition to explore the unknown, or actually believed India was not too far to the west of Spain. He certainly did call the inhabitants of the lands he discovered "Indians," a mislabeling which still persists.

But we do know that if the American continent had not existed, the experts would have been vindicated: Coumbus with his tiny ships could never have crossed an ocean as wide as the Atlantic and Pacific combined. In hindsight the exploration of the unknown may be justification enough!

As for the size of the Earth, it was accurately measured many times since (see item "geodesy" in an encyclopaedia), one notable effort being that of the French Academy of Sciences in the late 1700s. Their aim was to devise a new unit of distance, equal to one part in 10,000,000 of the distance from the pole to the equator (as Eratosthenes showed, it is enough to measure part of that distance). Nowadays that distance is known even more accurately, but the unit introduced by the French academy is still used as the standard of all distance measurements. It is called the meter.

"**Another look at Eratosthenes' and Posidonius' Determinations of the Earth's Circumference**" by Irene Fischer, *Quarterly Journal of the Royal Astronomical Society*, vol. 16, p. 152-167, 1975.

A delightful illustrated book for early readers about Eratosthenes, **The Librarian who Measured the Earth** by Kathryn Lansky (illustrated by Kevin Hawkes), Little Brown and Co., 1988, 1994.

Image Sources

(1) Alex Zaliznyak, David Stern. *Path of the Sun*. CC-BY-SA 3.0.

Chapter 17

Distance to the Horizon

Imagine you were standing at an elevation of h meters above the ocean and looking out across the water. What is the distance D to the horizon? It can be calculated, if you know the radius R of the Earth.

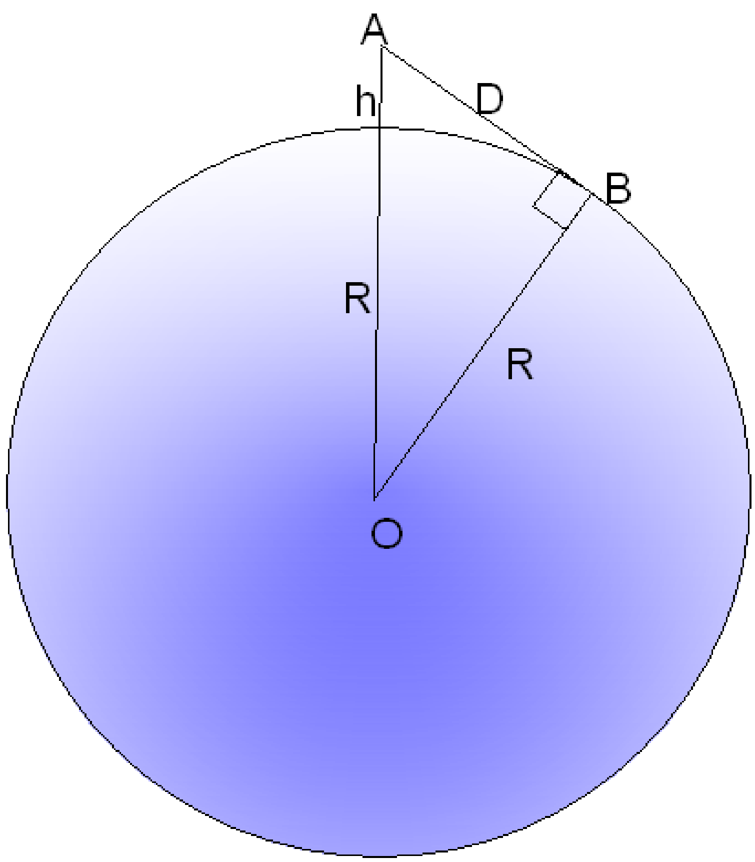

Figure 17.1: Illustration of the horizon calculation.

Your line of sight to the horizon is a **tangent** to the Earth — a line which touches the sphere of the Earth at just one point, marked B in the drawing here. If O is the center of the sphere of the Earth, by a well-known theorem of geometry such a tangent is perpendicular to the radius OB, that is, it makes a 90° angle with it.It follows from applying the Pythagorean theorem to $\triangle OAB$ that

$(OA)^2 = (AB)^2 + (OB)^2 or if the length of each line is spelled out, (R+h)^2 = D^2 + R^2 expanding the left side of the equation, we find R$
$2Rh + h^2 = D^2 + R^2 simplifying, we find 2Rh + h^2 = h(2R+h) = D^2$

Since the diameter $2R$ of the Earth is much bigger than h, we can replace $(2R+h)$ by $2R$ without siginficantly affecting the results. Carrying out this replacement gives

$$2\text{Rh} = \text{D}^2$$
$$D = \sqrt{2Rh}$$

This equation lets one calculate D — in kilometers, if h and R are given in kilometers. We can also rewrite is as

$$\text{D} = \sqrt{2Rh} = \sqrt{2R} \times \sqrt{h}$$

Using $R = 6371$ km, we find

$$\text{D} = 112.88 \ \text{ km} \times \sqrt{h}$$

If you are standing atop a mountain 1 km high, $h = 1$ km and your horizon should be 112.88 km away (we neglect the refraction of light in the atmosphere, which may modify this value). From the top of Mauna Kea on Hawaii, an extinct volcano about 4 km high (also the site of important astronomical observatories), the horizon should be about twice as distant, 226 km. On the other hand, standing on the beach with your eyes 2 meters = 0.002 km above the water, since $\sqrt{0.002} = 0.04472$, the horizon is only 5 km away.

The calculation should also hold the other way around. From a boat on the ocean you should begin seeing the top of Mauna Kea after you pass a distance of 226 km (again, not accounting for refraction). On November 15, 1806, Lieutenant Zebulon Pike of the US Army, leading an exploration party across the plains of the midwestern US, saw through his spyglass the top of a distant peak, just above the horizon. It took his party a week to cover the 100 miles to the mountain, which is now known as Pike's Peak, one of the tallest in Colorado. Pike actually tried to climb to its top, but the snow and the unexpected height of the mountain forced him back.

Image Sources

(1) David Shankbone. *Pike's Peak*. GNU Free Documentation.

(2) Alex Zaliznyak, David Stern. *Earth horizon.*. cc-by-sa-3.0.

Figure 17.2: Pike's Peak in Colorado.

Chapter 18

Parallax

18.1 Pre-Trigonometry

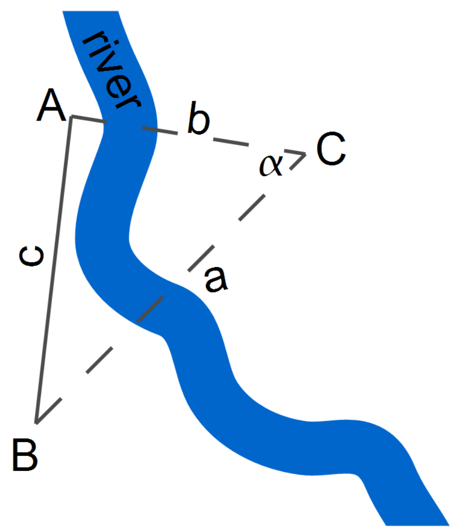

Figure 18.1: The basic problem of trigonometry.

The page at http://www.phy6.org/stargaze/Strig1.htm describes the basic problem of trigonometry (drawing above): finding the distance to some far-away point C, given the directions at which C appears from the two ends of a measured baseline AB. This problem becomes somewhat simpler if:

1. The baseline is **perpendicular** to the line from its middle to the object, so that $\triangle ABC$ is symmetric. We will denote its equal sides as $AB = BC = r$.
2. The length c of the baseline AB is much smaller than r. This means that the **angle** α between AC and BC is **small**; that angle is known as the **parallax** of C, as viewed from AB.
3. We do not ask for great accuracy, but are satisfied with an approximate value of the distance — say, within 1 per cent.

The method presented here was already used by the ancient Greeks more than 2000 years ago. They knew that the length of a circle of radius r was $2\pi r$, where π (a modern notation, not one of the Greeks, even though π is part of their alphabet) stands for a number a little larger than 3; approximately

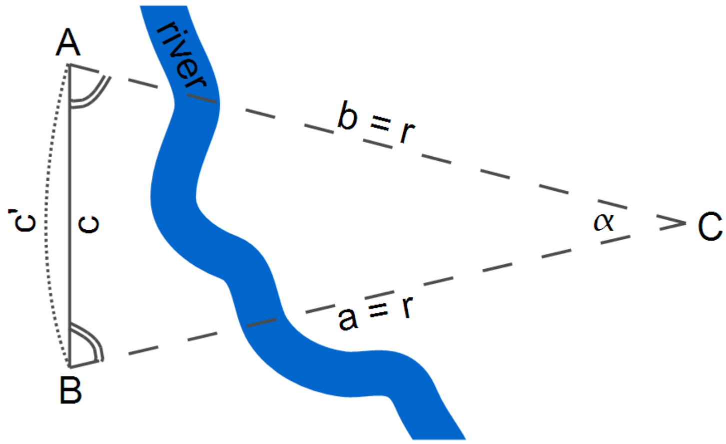

Figure 18.2: A simplified version of the problem (not to scale).

$\pi \approx 3.14159\ldots$

> (The Greek mathematician Archimedes derived π to about 4-figure accuracy, though he expressed it differently, since decimal fractions appeared in Europe only some 1000 years later.)

In this case (see **Figure** 18.2), we can approximate $\triangle ABC$ as a 'slice' of a much bigger circle; in this case, the length of the baseline is approximately equal to the length of the corresponding arc:

$\mathrm{c} \approx c'$

There are 360 degrees in a circle and α degrees in this particular arc; since 360 degrees corresponds to one circumference of arclength $(2\pi r)$, α degrees will correspond to an arclength of

$\mathrm{c'} = \frac{\alpha}{360^\circ \times 2\pi r}$

Solving for r and plugging in $c \approx c'$, we find

$\mathrm{r} = \frac{360^\circ}{2\pi\alpha \times c}$

We have solved for r in terms of c. For instance, if we know that $\alpha \approx 6^\circ$ (we'll see why this is relevant later), $2\pi\alpha = 36^\circ$ and we get:

$\mathrm{r} = 10\mathrm{c'}$

18.2 Estimating Distance Outdoors

Here is a method useful to hikers and scouts. Suppose you want to estimate the distance to some distant landmark — e.g. a building, tree or water tower.

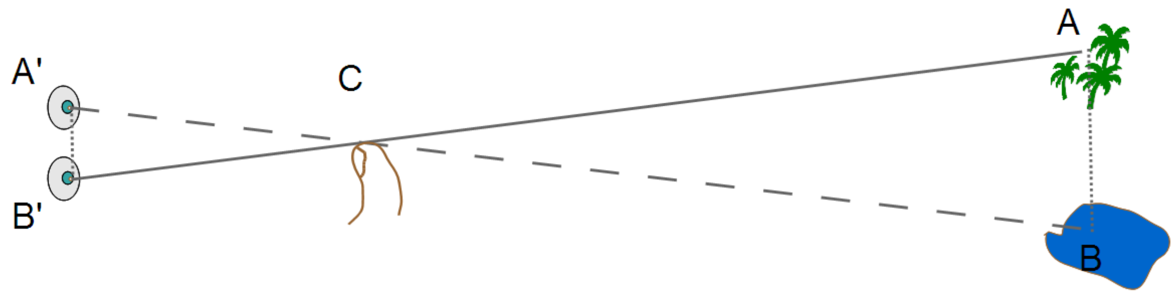

Figure 18.3: Finding the distance to a faraway point (A) (not to scale).

The drawing shows a schematic view of the situation from above (not to scale). To estimate the distance to the landmark A, you do the following:

1. **Stretch your arm forward and extend your thumb**, so that your thumbnail faces your eyes. Close one eye (A') and move your thumb so that, looking with your open eye (B'), you see your thumbnail covering the landmark A.
2. Then **open the eye you had closed (A') and close the one (B') with which you looked before**, without moving your thumb. It will now appear that your thumbnail has moved: it is no longer in front of landmark A, but in front of some other point at the same distance, marked as B in the drawing.
3. **Estimate the true distance AB**, by comparing it to the estimated heights of trees, widths of buildings, distances between power-line poles, lengths of cars etc. The distance to the landmark is **10 times** the distance AB.

Why does this work? Because even though people vary in size, the proportions of the average human body are fairly constant, and for most people, the angle between the lines from the eyes (A',B') to the outstretched thumb is about 6 degrees, for which the ratio 1:10 was found in an earlier part of this section. That angle is the parallax of your thumb, viewed from your eyes. The triangle A'B'C has the same proportions as the much larger triangle ABC, and therefore, if the distance B'C to the thumb is 10 times the distance A'B' between the eyes, the distance AC to the far landmark is also 10 times the distance AB.

18.3 How far to a Star?

When estimating the distance to a very distant object, our "baseline" between the two points of observation better be large, too. The most distant objects our eyes can see are the stars, and they are very far indeed: light which moves at 300,000 kilometers (186,000 miles) per second, would take years, often many years, to reach them. The Sun's light needs 500 seconds to reach Earth, a bit over 8 minutes, and about 5.5 hours to reach the average distance of Pluto, the most distant planet. A "light year" is about 1600 times further, an enormous distance.

The biggest baseline available for measuring such distances is the diameter of the Earth's orbit, 300,000,000 kilometers. The Earth's motion around the Sun makes it move back and forth in space, so that on dates separated by half a year, its positions are 300,000,000 kilometers apart. In addition, the entire solar system also moves through space, but that motion is not periodic and therefore its effects can be separated.

And how much do the stars shift when viewed from two points 300,000,000 km apart? Actually, very, very little. For many years astronomers struggled in vain to observe the difference. Only in 1838 were definite parallaxes measured for some of the nearest stars — for Alpha Centauri by **Henderson** from South Africa, for Vega by **Friedrich von Struve** and for 61 Cygni by **Friedrich Bessel**.

Such observations demand enormous precision. Where a circle is divided into 360 degrees (360°), each degree is divided into 60 minutes (60')–also called "minutes of arc" to distinguish them from minutes of time–and each minute contains 60 seconds of arc (60"). All observed parallaxes are less than 1", at the limit of the resolving power of even large ground-based telescopes.

In measuring star distances, astronomers frequently use the parsec, the distance to a star whose yearly parallax is 1" — one second of arc. One parsec equals 3.26 light years, but as already noted, no star is that close to us. Alpha Centauri, the sun-like star nearest to our solar system, has a distance of 4.3 years and a parallax of 0.75".

Alpha Centauri is not a name, but a designation. Astronomers designate stars in each constellation by letters of the Greek alphabet — alpha, beta, gamma, delta and so forth, and "Alpha Centauri" means the brightest star in the constellation of Centaurus, located high in the southern skies. You need to be south of the equator to see it well.

Image Sources

(1) Alex Zaliznyak. *Trigonometry Problem 2*. cc-by-sa-3.0.

(2) Alex Zaliznyak. *Trigonometry Problem*. cc-by-sa-3.0.

(3) Alex Zaliznyak. *Trigonometry Problem 3*. cc-by-sa-3.0.

Chapter 19

Estimating the Distance to the Moon

19.1 Aristarchus' Method

Aristarchus around 270 BC derived the Moon's distance from the duration of a **lunar eclipse** (Hipparchus later found an independent method).

It was commonly accepted in those days that the Earth was a sphere (although its size was only calculated a few years later, by Eratosthenes (See the chapter "The Round Earth and Columbus" in the *From Stargazers to Starships FlexBook* on www.ck12.org). Astronomers also believed that the Earth was the center of the universe, and that Sun, Moon, planets and stars all orbited around it. It was only natural, then, that Aristarchus assumed that the Moon moved in a **large circle** around Earth.

Let R be the radius of that circle and T the time it takes the Moon to go around once, about one month. In that time the Moon covers a distance of $2\pi R$, where $\pi \approx 3.1415926\ldots$ (pronounced "pi") is a mathematical constant, the ratio of the circumference of any circle to its diameter.

An eclipse of the Moon occurs when the Moon passes through the shadow of the Earth, on the opposite side from the Sun (therefore, it must be a **full** Moon). If r is the radius of the Earth, the shadow's width is close to the Earth's diameter, or$2r$. Let t be the time it takes the mid-point of the Moon to cross the **center** of the shadow, about 3 hours (in eclipses of the longest duration, when the Moon crosses the center of the shadow).

If the moon moves around the Earth at a constant speed — and it takes time T (again, about a month) to cover $2\pi R \approx 6.28R$, its speed can be expressed as the ratio of distance traveled to the time it takes as

V_m = 2 $\pi R_{\overline{T}}$

Since it takes the moon about 3 hours to travel a distance of $2r$, we can also express its speed as:

V_m = 2 r$_{\bar{t}}$

Setting these equal to each other, we find:

$$6.28\ \mathrm{R}_{\overline{2r=\frac{T}{t}}}$$

From this Aristarchus obtained

$$\mathrm{R}_{\overline{r\approx 60}}$$

This result fits the average distance of the Moon accepted today, 60 Earth radii.

19.2 A Few Extra Details

The word "about" was used here more than once. For instance, the orbital period of the Moon was stated to be "about" one month. In fact, the length of the "lunar month" from one **new Moon** to the next (or from one **full Moon** to the next) is 29.53 days, but the Moon's orbital period is actually 2.21 days shorter (this is discussed in the section on the calendar (See the chapter "The Calendar" in the *From Stargazers to Starships FlexBook* on www.ck12.org).

Viewed from Earth, a "new Moon" (occurring between the time a thin crescent is last seen **before** sunrise and the time one is seen shortly **after** sunset) happens when the Moon in its apparent motion around the sky overtakes the Sun. However, by the time of the **next** new Moon, the Sun's position in the sky has already shifted. If the Sun takes 12 months to go around the sky (or around the ecliptic, or around the zodiac), then in one month it completes 1/12 of its circuit. The Moon must therefore complete [1+(1/12)] circuits to catch up with the Sun again, and the lunar month ("synodic period") is about 1/12 of a month longer than the actual period of 27.32 days.

Also, the Earth's shadow has only approximately the width of $2r$. It would have very nearly a width of $2r$ if the Sun were a point-like light source (**exactly** that width if it were infinitely far away). Actually, however, the Sun is large enough to appear as a **disk** which covers about half a degree of the sky. As a result, the Earth's shadow is not a cylinder but a gradually narrowing cone, and at the Moon's distance it is already about 25% narrower than $2r$.

Here is another way of looking at the same process. Suppose we observe the eclipse from the Moon. Seen from there, the Earth moves from east to west — from A to B in the drawing, assuming the eclipse is of greatest length (i.e. the **middle** of the Earth passes in front of the Sun).

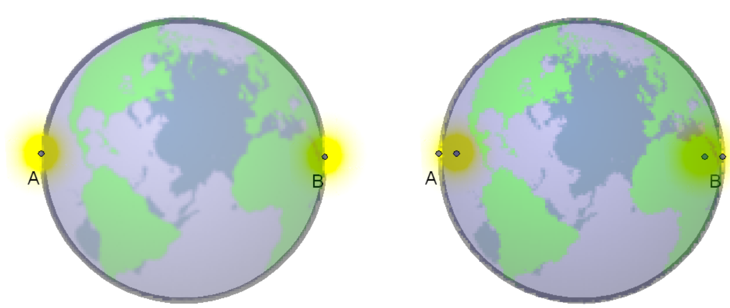

Figure 19.1: Earth blocking the Sun during a lunar eclipse (not to scale).

The eclipse begins when the **last** bit of the western edge of the Sun passes point A (right of the drawing shown) and ends when the **first** bit of the eastern edge of the Sun pokes out at point B. That takes less time than it takes for the **center** of the Sun to pass from A to B (left drawing) which would be the duration of the eclipse if the Sun were a tiny point source, located at its middle.

Image Sources

(1) Alex Zaliznyak. *Lunar Eclipse*. cc-by-sa-3.0.

Chapter 20

Distance to the Moon, Part 2

Hipparchus, who used an eclipse of the Moon to deduce the precession of the equinoxes (See the chapter "Precession" in the *From Stargazers to Starships FlexBook* on www.ck12.org), used a **total eclipse of the Sun** — probably in 129 BC–to estimate how far the Moon was. That distance had also been derived from a lunar eclipse by Aristachus–see the chapter "Estimating the Distance to the Moon" in the *From Stargazers to Starships FlexBook* on www.ck12.org.

That eclipse was **total** at the Hellespont — the Dardanelles, part of the narrow strait that separates the European and Asian parts of Turkey — but only **4/5 of the Sun were covered** in Alexandria of Egypt, further to the south.

Hipparchus knew that when the Sun was eclipsed, it and the Moon occupied the same spot on the sphere of the heavens. The reason, he assumed, was that the Moon passed between us and the Sun.

He believed that the Sun was much more distant than the Moon, as Aristarchus of Samos had concluded, about a century earlier, from observing the time when the Moon was exactly half full (see chapters "Estimating the Distance to the Moon" and "Does the Earth Revolve Around the Sun" in the *From Stargazers to Starships FlexBook* on www.ck12.org). He also assumed that the peak of the eclipse occurred at the same time at both locations (not assured, but luckily not too far off), and he then carried out the following calculation.

20.1 The Eclipse

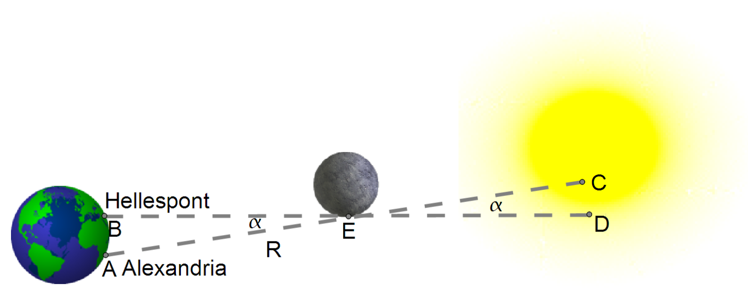

Figure 20.1: Moon blocking the Sun during a solar eclipse (not to scale).

In a total eclipse of the Sun, the Moon just barely covers the Sun. The Sun itself is so distant that when viewed from anywhere on Earth, it covers practically the same patch of the sky, with a width of about 0.5

degrees. Hipparchus concentrated on point E at the edge of the Moon (drawing), which during totality, when **viewed from the Hellespont** (point B) just overlapped point D on the edge of the Sun.

Viewed from Alexandria (point A), at that same moment, the point E only overlapped point C on the Sun, about 1/5 solar diameter short of the edge — which was why the eclipse there was not total. One-fifth of the Sun's diameter covers about 0.1 degrees in the sky, so the small angle α (alpha, Greek A) between the two directions measured about 0.1 degrees. That angle is the parallax (See the chapter "Parallax" in the *From Stargazers to Starships FlexBook* on www.ck12.org) of the edge of the Moon, viewed from the above two locations.

It is unlikely that Hipparchus knew the distance AB, but he probably knew the latitudes of the Hellespont and Alexandria. The local latitude can be shown to be equal to the elevation of the celestial pole above the horizon and today can be readily deduced by observing the height of Polaris, the pole star above the horizon. In the time of Hipparchus the pole of the heavens wasn't near Polaris (because of the precession of the equinoxes), but Hipparchus, who had mapped the positions of about 850 stars, must have known its position quite well.

The latitude of the Hellespont (from a modern atlas) is about 40°20′ (40 degrees and 20 minutes, 60 minutes per degree), while that of Alexandria is about 31°20′, a difference of 9 degrees. We will also assume Alexandria is exactly due south. If furthermore r is the radius of the Earth, then the circumference of the Earth is $2\pi r$. Since the circumference also spans 360 degrees, we get

$$\mathrm{AB} = 2\,\pi r \frac{}{360\times 9}$$

20.2 The Distance to the Moon

The points AB are also located on another circle, centered on the Moon. The radius in that case is the distance R to the Moon, and because the arc AB covers 0.1 degrees, we get

$$\mathrm{AB} = 2\,\pi r \frac{}{360\times 0.1}$$

Strictly speaking, each of the two arcs AB expressed in the above equations is measured along a different circle, with a different radius (and the two circles curve in opposite ways). However, in both cases AB covers only a small part of the circle, so that as an approximation we may regard each of the arcs as equal to the straight-line distance AB. That assumption allows us to regard the two expressions as equal and to write

$$2\,\pi r \frac{}{360\times 0.1 = \frac{2\pi r}{360}\times 9}$$

Multiplying both sides by 360 and dividing by 2π give

$$0.1\ \mathrm{R} = 9\ \mathrm{r} \implies \frac{R}{r} = 90$$

suggesting the Moon's distance is 90 Earth radii, an overestimate of about 50 per cent.

20.3 A more accurate calculation

One reason an excessive value was obtained is that the Moon was assumed to be **overhead** at A or B. Actually, it is likely to be at some significant angle to the overhead direction, the "zenith" (see drawing).

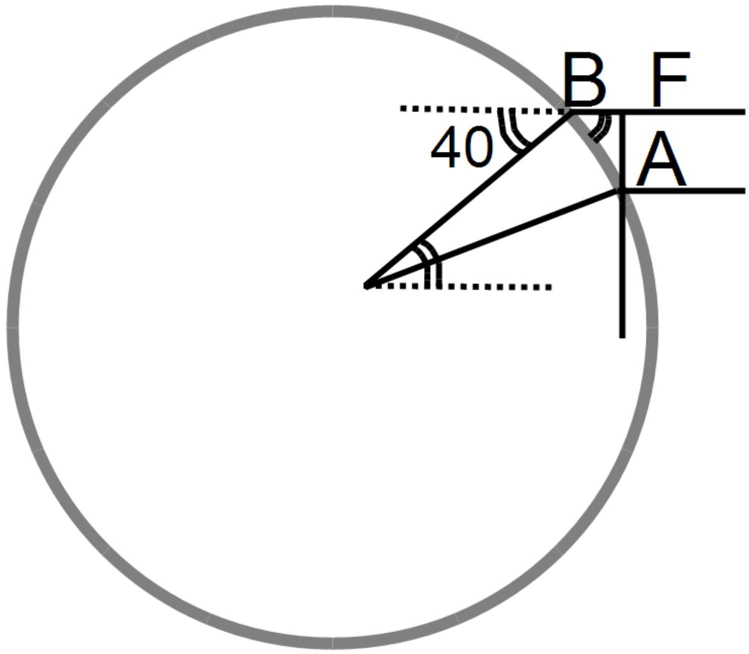

Figure 20.2: Close up of the eclipse.

Then the section cut by the angle α from the circle of radius R around E is not AB but AF (second drawing), which is smaller. Taking this into account reduces the distance.

We don't know where the Sun was during the 129 BC eclipse, but it must have been on the ecliptic (the words are obviously related!), which places it within 23.5° of the celestial equator, on either side. Assuming it was **on the equator** (that is, it passed overhead on the Earth's equator) and south of the reported observations (i.e. the eclipse occurred **near noon**) one can make a crude estimate of the correction, using simple trigonometry (see http://www.phy6.org/stargaze/Strig2.htm).

The Hellespont is around latitude of 40 degrees, and as the drawing shows, that is also the angle between the Moon's direction and the zenith. From the drawing:

$$\text{AF} = \text{AB} \cos 40^\circ = 0.766AB$$

Repeating the preceding calculation for AF:

$$\text{AF} = 2\pi R_{\overline{360\times 0.1AF=0.766AB=0.766\times\frac{2\pi r}{360}\times 9 Andintheend,\frac{R}{r}=90\times 0.766=69}}$$

20.4 Final comments

According to "A History of Astronomy" by A. Pannekoek, the result obtained by Hipparchus was **between 62 and 73 Earth radii**. Today we know the average distance is about 60 radii, varying by a few Earth radii either way because of the ellipticity of the Moon's orbit.

In the absence of accurate timing, the method is **almost guaranteed to produce an overestimate**. The Earth rotates beneath the shadow spot cast by the Moon, which makes that spot sweep over a long strip, hitting many different locations at different times. The Hellespont was just one of many places where the eclipse was total. Similarly, Alexandria was just one of many locations where 4/5 of the Sun was covered. Randomly selecting point B from the first group and point A from the second may give a much longer baseline AB and a much larger (and incorrect) distance of the Moon. The fact Alexandria is almost exactly south of the Hellespont does not guarantee their peak eclipse times are the same, just that they are not too different.

Image Sources

(1) Alex Zaliznyak. *Solar Eclipse 2*. cc-by-sa-3.0.

(2) Alex Zaliznyak. *Solar Eclipse*. cc-by-sa-3.0.

Chapter 21

Does the Earth Revolve Around the Sun

Aristarchus of Samos, an early Greek astronomer (about 310 to 230 BC), was the first to suggest that the Earth revolved around the Sun, rather than the other way around. He gave the first estimate of the distance of the Moon (See the chapter "Estimating the Distance to the Moon" in the *From Stargazers to Starships FlexBook* on www.ck12.org), and it was his careful observation of a lunar eclipse–pin-pointing the Sun's position on the opposite side of the sky–that enabled Hipparchus, 169 years later, to deduce the precession of the equinoxes (See the chapter "Precession" in the *From Stargazers to Starships FlexBook* on www.ck12.org).

Except for one calculation — an estimate of the distance and size of the Sun — no work of Aristarchus has survived. However, one could guess why he believed that the Sun, not the Earth, was the central body around which the other one revolved. His calculation suggested that **the Sun was much bigger** than the Earth — a watermelon, compared to a peach — and it seemed unlikely that the larger body would orbit one so much smaller.

Here we will develop a line of reasoning somewhat like the one Aristarchus used (for his actual calculation, see reference at the end). Aristarchus started from an observation of a **lunar eclipse** (See the chapter "Estimating the Distance to the Moon" in the *From Stargazers to Starships FlexBook* on www.ck12.org). At such a time the Moon moves through the Earth's shadow, and what Aristarchus saw convinced him that the shadow was about **twice as wide** as the Moon. Suppose the width of the shadow was also the width of the Earth (actually it is less — see below, also here). Then the diameter of the Moon would be **half the Earth's**.

Aristarchus next tried to observe **exactly when half the moon was sunlit**. For this to happen, the angle Earth-Moon-Sun ($\angle EMS$ in the drawing here) must be exactly **90 degrees**.

Knowing the Sun's motion across the sky, Aristarchus could also locate the point P in the sky, on the Moon's orbit (near the ecliptic), which was exactly 90 degrees from the direction of the Sun **as seen from Earth**. If the Sun were very, **very** far away, the half-moon would also be on this line, at a position like M' (drawn with a different distance scale, for clarity).

Aristarchus estimated, however, that the direction to the half-Moon made a **small angle** a with the direction to P, about 1/30 of a right angle or 3 degrees.

As the drawing shows, $\angle EMS$ (Earth-Moon-Sun) then also must equal 3 degrees. If R_s is the Sun's distance and R_m the Moon's, a full 360 degree circle around the Sun at the Earth's distance has a length of $2\pi R_s$. The distance $R_m = EM$ is then about as long as an **arc** of that circle covering 3 degrees, or 1/120 of the

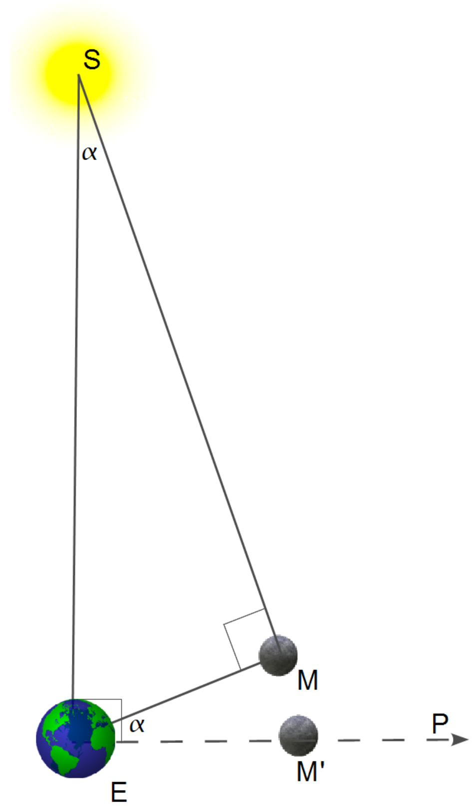

Figure 21.1: Illustration of Aristarchus' calculation.

full circle. It follows that:

R_m $= 2\ \pi R_{s\,\overline{120\approx\frac{R_s}{19}}}$

Therefore,

R_s$_{\overline{R_m\approx 19,}}$

making the Sun — according to Aristarchus — 19 times more distant than the Moon. Since the two have very nearly the same size in the sky — even though one of them is 19 times more distant, the Sun must also be 19 times larger in diameter than the Moon.

If the Moon's diameter is **half** the size of the Earth's, the Sun must be 19/2 or nearly 10 times wider than the Earth. The effect described in the figures of the next section modifies this argument somewhat (`http://www.phy6.org/stargaze/Sshadow.htm`), making the Earth 3 times wider than the Moon, not twice. **If Aristarchus had observed correctly**, that would make the Sun's diameter 19/3 times — a bit more than 6 times — than the Earth.

Actually, he had not! His method does not really work, because in actuality the position of the half-Moon is very close to the line OP. The angle α, far from being 3 degrees, is actually so small that Aristarchus could never have measured it, especially without a telescope. The actual distance to the Sun is about 400 times that of the Moon, not 19 times, and the Sun's diameter is similarly about 400 times the Moon's and more than 100 times the Earth's.

But it makes no difference. The main conclusion, that the Sun is vastly bigger than Earth, still holds. Aristarchus could just as well have said that the angle α was at **most 3** degrees, in which case the Sun was at least 19 times more distant than the Moon, and its size **at least** 19/3 times that of Earth. In fact he **did** say so — but he also claimed it was **less than 43/6** times larger than the Earth (Greeks used simple fractions–they knew nothing about decimals), which was widely off the mark. But it makes no difference: as long as the Sun is much bigger than the Earth, it makes more sense that it, rather than the Earth, is at the center.

Good logic, but few accepted it, not even Hipparchus and Ptolemy. In fact, **the opposite argument** was made: if the Earth orbited the Sun, it would be on opposite sides of the Sun every 6 months. If that distance was as big as Aristarchus claimed it to be, would not the positions of the stars differ when viewed from two spots so far apart? We now know the answer: the stars are so far from us, that even with the two points 20 times further apart than Aristarchus had claimed, the best telescopes can barely observe the shift of the stars. It took nearly 18 centuries before the ideas of Aristarchus were revived by Copernicus.

Chapter 22

The Earth's Shadow

In a lunar eclipse, if the width of the shadow of the Earth is twice the width of the Moon, then the width of the Earth itself is (very nearly) three times that of the Moon — not twice, as one might perhaps think. Here is why:

The Sun is not a point of light but an extended source, with a disk covering a circular patch in the sky, about 0.5 degrees across. This makes the shadow of the Earth not a cylinder, stretching to infinity without narrowing down, but a cone, with an angle of 0.5 degrees across its apex C (drawing). AB is here the diameter of the Earth, and the directions AC and BC represents rays from opposite edges of the Sun's disk, rays whose directions differ by 0.5 degrees.

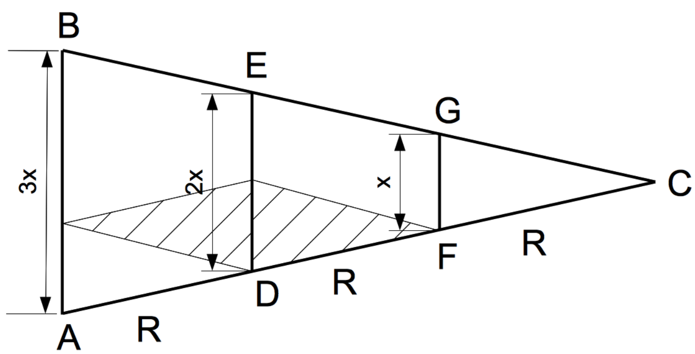

Figure 22.1: Illustration of a Lunar eclipse.

If x is the diameter of the Moon and R its distance, then according to Aristarchus, the width ED of the shadow at distance Requals $2x$ (actually, $2.5x$ comes closer to the mark). We add to the drawing points H and K so that HA = KD = x.

The width of the Moon as seen from point H is KD = x, and since the Moon's size in the sky is about the same as the Sun's, the angle KHD (shaded) should also equal 0.5 degrees. We now extend the line AD = R a further distance R to point F. Then the two shaded triangles HKD and KFD are congruent (= same in size and shape) and have the same 0.5 angle as the angle at C. Indeed, one can prove now that the triangles GFC and AHD are also congruent to the two shaded ones.

It follows then that AC = $3R$, and from simple proportions (see drawing) AB = $3x$.

Image Sources

(1) David Stern. *Earth Shadow.*. CC-BY-SA 3.0.

Chapter 23

The Planets

Most stars we observe form fixed constellations in the sky, undergoing daily motion (e.g. rising and setting) but maintaining fixed positions relative to each other — like the stars of Orion, or the Big Dipper. The ancients however noted that 5 stars constantly moved — all following close to the paths of the Sun and Moon across the heavens, i.e., close to the ecliptic. The Greeks called them planets, Greek for wanderers, a name still used.

23.1 Venus and Mercury

The five planets known to the ancients were named after principal Greek gods, later replaced by their Roman equivalents: **Mercury, Venus, Mars, Jupiter and Saturn**. They were relatively bright — Venus and Jupiter can be brighter than any fixed star — though their brightness seemed to vary. Venus and Mercury never appear far from the Sun and (outside the polar regions, at least) are only visible just after sunset or before sunrise, suggesting that those planets were confined near the Sun. The Greeks called Venus "Hesperus" when it appeared as the evening star and "Phosphorus" when as morning star it rose before sunrise, though they realized both were the same object. Mercury, which is fainter and closer to the Sun, is particularly hard to detect by eye and this only when its visible position is far from the Sun's.

All planets seemed to move among the stars in the **same direction as the Moon** (and of the Sun) — with one strange variation: sometimes their apparent motion is temporarily reversed ("retrograde motion"). That is most evident with Mercury and Venus, which shuttle back and forth across the position of the Sun. As the Sun moved among the stars — along the constellations of the zodiac — these planets sometimes move the same way and add their motion to that of the Sun, but sometimes their apparent motion opposes the one of the Sun, causing them to seem to move backwards or "retrograde."

23.2 Mars and Jupiter

The other three planets visible to the eye can be seen anywhere along the ecliptic–even at midnight, directly opposite the Sun, which was when they appear brightest. Mars seems to move the fastest, Jupiter next, and Saturn the slowest. But all exhibit that puzzling quirk–near the point of their celestial path exactly opposite the Sun ("opposition"), their motion among the stars temporarily turns around.

Today we undestand all that very well (see image above). Planets are spherical objects like Earth — Venus, Mercury and Mars are smaller, Jupiter and Saturn much bigger. Earth is a planet too and others exist as well (too faint to be seen without a telescopes), all orbiting the sun on or near the plane of the ecliptic. Their

Figure 23.1: Retrograde motion of Mars (big circles) and Uranus (small circles).

speed however varies–the closer to the Sun, the faster (see http://www.phy6.org/stargaze/Skeplaws.htm and in particular Kepler's third law). Therefore, when the three outer planets are near opposition, the Earth orbiting closer to the Sun overtakes them, and they seem to move backwards.

The retrograde motion of the two **inner** planets has a similar cause. Being closer to the Sun, they **overtake the Earth** in their motion.

23.3 Components of the Solar System

Here is a quick summary of the components of the solar system–besides the fact that all of them orbit the Sun, including Earth and two major planets too dim for ancient astronomers to have seen. Four distinct classes of objects are usually recognized:

1. Major planets, in order of distance from the Sun—**Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune**. All but the inner two have satellites, and all four outer ones have rings as well, composed of small orbiting chunks of matter.
2. **Asteroids** or minor planets, most but not all between Mars and Jupiter. Ranging in diameter up to about 500 km.
3. The "**Kuiper Belt**" of icy objects outside the orbit of Neptune, of which the best known (though as of now only the second largest) is **Pluto**, discovered in 1930 and about the size of our Moon. The belt is named after the Belgian astronomer Gerard Kuiper, may extend to twice the distance of Neptune and is estimated to consist of as many as 100,000 objects (about 1000 of them identified so far), many only 100 km across or smaller.
4. **Comets**, traditionally divided into "non-returning" (official name, "long period comets") and "periodic" ones. **Non-returning** comets are believed to come from the "Oort cloud," a huge near-spherical collection of frozen chunks on the distant fringes of the solar system. They are loosely bound to the Sun, and now and then the gravity of a distant star is believed to slightly change the motion of some and send them sunwards. They become visible as comets when sunlight evaporates some of their surface to create the comet's glow and tail. **Periodic** comets were once believed to have started as non-returning ones but to have been diverted by the pull of one of the larger planets. They are now widely held to arrive from the Kuiper belt as a class of objects known as **Centaurs**.

23.4 Early History, False Leads

As noted earlier (See the chapter "Does the Earth Revolve Around the Sun" in the *From Stargazers to Starships FlexBook* on www.ck12.org), Aristarchus of Samos proposed that the Earth revolved around the Sun, but the idea was rejected by later Greek astronomers, in particular by Hipparchus. Ptolemy, living in Egypt in the 2nd century AD, expressed the consensus when he argued that all fixed stars were on some distant sphere which rotated around the Earth. Ptolemy tried to assemble and write down all that was known in his day about the heavens in "The Great Treatise," now known as the "Almagest," a corruption of its Arab name. (An annotated translation by G.J. Toomre was published in 1984 by Princeton University Press and is now available in paperback for $39.50. See p. 120, Nature vol. 397, 14 January 1999.)

To explain the motion of planets, Ptolemy used a theory which started with Hipparchus. Following the work of Aristarchus (See the chapter "Estimating the Distance to the Moon" in the *From Stargazers to Starships FlexBook* on www.ck12.org)and Hipparchus (See the chapter "Distance to the Moon, Part 2" in the *From Stargazers to Starships FlexBook* on www.ck12.org), it was already accepted that the Moon moved around Earth. Ptolemy assumed that the Sun, planets and the distant stars (whatever those were) also moved around the Earth. To the Greeks, the circle represented perfection, and Ptolemy assumed Moon, Sun and stars moved in circles too. Since the motion was not exactly uniform (later explained by Kepler's laws — http://www.phy6.org/stargaze/Skeplaws.htm), he assumed that these circles were centered some distance away from the Earth.

While the Sun moved around Earth, Venus and Mercury obviously moved around it, on circles of their own, centered near the Sun. But what about Mars, Jupiter and Saturn? Cleverly, Ptolemy proposed that like Venus and Mercury, each of them also rotated around a point in the sky that orbited around Earth like the Sun, except that those points were empty. The backtracking of the planets now looked similar to the backtracking of Venus and Mercury. The center carrying each of those planets accounted for the planet's regular motion, but to this the planet's own motion around that center had to be added, and sometimes the sum of the two made the planet appear (for a while) to advance backwards.

This "explanation" left open the question **what** the planets, Sun and Moon were, and **why** they displayed such strange motions. But worse, it was also inaccurate. As the positions of the planets were measured more and more accurately, additional corrections had to be introduced.

Yet Ptolemy's view of the solar system dominated European astronomy for over 1000 years. One reason was that astronomy almost stopped in its tracks during the decline and fall of the Roman Empire and during the "dark ages" that followed. The study of the heavens continued in the Arab world, under Arab rulers, but of all the achievements of Arab astronomers, the one which exerted the greatest influence was the preservation and translation of Ptolemy's books, and thus of his erroneous views.

Image Sources

(1) Tunc Tezel. *Retrograde Mars*. Public Domain.